8. ***REASONING*** The period T of a satellite is the time for it to make one complete revolution around the planet. The period is the circumference of the circular orbit ($2\pi R$) divided by the speed v of the satellite, so that $T = (2\pi R)/v$ (see Equation 5.1). In Section 5.5 we saw that the centripetal force required to keep a satellite moving in a circular orbit is provided by the gravitational force. This relationship tells us that the speed of the satellite must be $v = \sqrt{GM/R}$ (Equation 5.5), where G is the universal gravitational constant and M is the mass of the planet. By combining this expression for the speed with that for the period, and using the definition of density, we can obtain the period of the satellite.

SOLUTION The period of the satellite is

$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{\dfrac{GM}{R}}} = 2\pi\sqrt{\frac{R^3}{GM}}$$

According to Equation 11.1, the mass of the planet is equal to its density ρ times its volume V. Since the planet is spherical, $V = \frac{4}{3}\pi R^3$. Thus, $M = \rho V = \rho\left(\frac{4}{3}\pi R^3\right)$. Substituting this expression for M into that for the period T gives

$$T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R^3}{G\rho\left(\frac{4}{3}\pi R^3\right)}} = \sqrt{\frac{3\pi}{G\rho}}$$

The density of iron is $\rho = 7860\ \text{kg/m}^3$ (see Table 11.1), so the period of the satellite is

$$T = \sqrt{\frac{3\pi}{G\rho}} = \sqrt{\frac{3\pi}{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(7860\ \text{kg/m}^3\right)}} = \boxed{4240\ \text{s}}$$