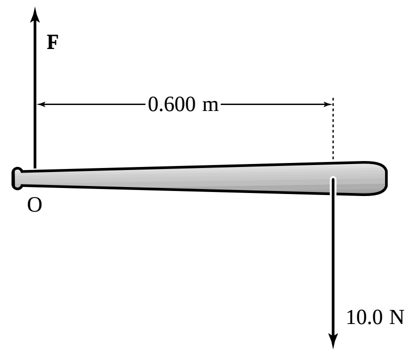
Chapter 12 Solutions

12.1 To hold the bat in equilibrium, the player must exert both a force and a torque on the bat to make



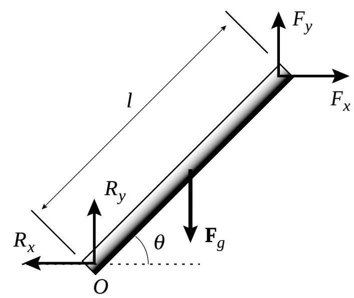
$$\sum F_x = \sum F_y = 0 \quad \text{and} \quad \sum \tau = 0$$

$\sum \mathbf{F_y} = 0 \Rightarrow F - 10.0\text{ N} = 0$, or the player must exert a net upward force of $F = \boxed{10.0\text{ N}}$

To satisfy the second condition of equilibrium, the player must exert an applied torque τ_a to make $\sum \tau = \tau_a - (0.600\text{ m})(10.0\text{ N}) = 0$. Thus, the required torque is

$$\tau_a = +6.00\text{ N}\cdot\text{m} \quad \text{or} \quad \boxed{6.00\text{ N}\cdot\text{m counterclockwise}}$$

12.2 Use distances, angles, and forces as shown. The conditions of equilibrium are:

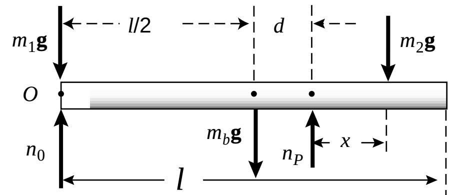


$$\sum F_y = 0 \Rightarrow \boxed{F_y + R_y - F_g = 0}$$

$$\sum F_x = 0 \Rightarrow \boxed{F_x - R_x = 0}$$

$$\sum \tau = 0 \Rightarrow \boxed{F_y\, l \cos\theta - F_g\left(\frac{l}{2}\right)\cos\theta - F_x\, l \sin\theta = 0}$$

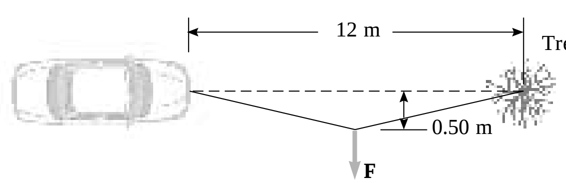
12.3 Take torques about P.



$$\sum \tau_p = -n_0\left[\frac{l}{2} + d\right] + m_1 g\left[\frac{l}{2} + d\right] + m_b g d - m_2 g x = 0$$

We want to find x for which $n_0 = 0$.

$$x = \frac{(m_1 g + m_b g)d + m_1 g l/2}{m_2 g} = \boxed{\frac{(m_1 + m_b)d + m_1 l/2}{m_2}}$$



12.4 $\tan\alpha = \dfrac{0.500}{6.00}$

$\alpha = 4.76°$

$F = 2T\sin\alpha$

$$T = \frac{F}{2 \sin \alpha} = \boxed{3.01 \text{ kN}}$$

***12.5** The location of the center of gravity is defined as

$$x_{CG} \equiv \frac{\sum_{i=1}^{n} m_i g_i x_i}{\sum_{i=1}^{n} m_i g_i}$$

If the system is in a uniform gravitational field, this reduces to

$$x_{CG} \equiv \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}$$

Thus, for the given two particle system:

$$x_{CG} = \frac{(3.00\text{ kg})(-5.00\text{ m}) + (4.00\text{ kg})(3.00\text{ m})}{3.00\text{ kg} + 4.00\text{ kg}} = \boxed{-0.429\text{ m}}$$

12.6 The hole we can count as negative mass

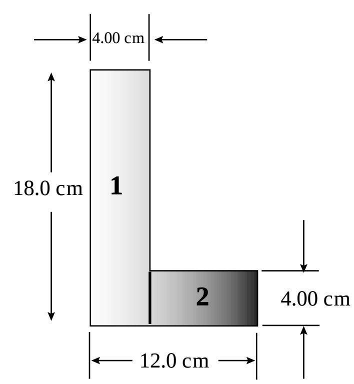
$$x_{CG} = \frac{m_1 x_1 - m_2 x_2}{m_1 - m_2}$$

Call σ the mass of each unit of pizza area.

$$x_{CG} = \frac{\sigma \pi R^2\, 0 - \sigma \pi (R/2)^2\,(-R/2)}{\sigma \pi R^2 - \sigma \pi (-R/2)^2}$$

$$x_{CG} = \frac{R/8}{3/4} = \boxed{\frac{R}{6}}$$

12.7 The coordinates of the center of gravity of piece 1 are



$$x_1 = 2.00\text{ cm} \quad \text{and} \quad y_1 = 9.00\text{ cm}$$

The coordinates for piece 2 are

$$x_2 = 8.00\text{ cm} \quad \text{and} \quad y_2 = 2.00\text{ cm}$$

The area of each piece is

$$A_1 = 72.0\text{ cm}^2 \quad \text{and} \quad A_2 = 32.0\text{ cm}^2$$

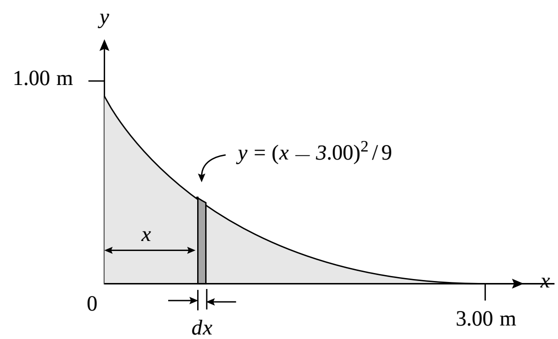
And the mass of each piece is proportional to the area. Thus,

$$x_{CG} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(72.0\ \text{cm}^2)(2.00\ \text{cm}) + (32.0\ \text{cm}^2)(8.00\ \text{cm})}{72.0\ \text{cm}^2 + 32.0\ \text{cm}^2} = \boxed{3.85\ \text{cm}}$$

and

$$y_{CG} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(72.0\ \text{cm}^2)(9.00\ \text{cm}) + (32.0\ \text{cm}^2)(2.00\ \text{cm})}{104\ \text{cm}^2} = \boxed{6.85\ \text{cm}}$$

12.8 Let σ represent the mass-per-face area. A vertical strip at position x, with width dx and height $(x - 3.00)^2/9$ has mass



$$dm = \sigma(x - 3.00)^2 dx/9$$

The total mass is

$$M = \int dm = \int_{x=0}^{3.00} \sigma(x-3)^2 dx/9$$

$$M = (\sigma/9)\int_0^{3.00}(x^2 - 6x + 9)dx$$

$$M = \left(\frac{\sigma}{9}\right)\left[\frac{x^3}{3} - \frac{6x^2}{2} + 9x\right]_0^{3.00} = \sigma$$

The x-coordinate of the center of gravity is

$$x_{CG} = \frac{\int x\,dm}{M} = \frac{1}{9\sigma}\int_0^{3.00}\sigma\,x(x-3)^2\,dx = \frac{\sigma}{9\sigma}\int_0^{3.00}(x^3 - 6x^2 + 9x)\,dx$$

$$x_{CG} = \frac{1}{9}\left[\frac{x^4}{4} - \frac{6x^3}{3} + \frac{9x^2}{2}\right]_0^{3.00} = \frac{6.75\ \text{m}}{9.00} = \boxed{0.750\ \text{m}}$$

12.9 Let the fourth mass (8.00 kg) be placed at (x, y), then

$$x_{CG} = 0 = \frac{(3.00)(4.00) + m_4(x)}{12.0 + m_4}$$

$$x = -\frac{12.0}{8.00} = \boxed{-1.50\ \text{m}}$$

Similarly, $y_{CG} = 0 = \dfrac{(3.00)(4.00) + 8.00(y)}{12.0 + 8.00}$

$$y = \boxed{-1.50\ \text{m}}$$

***12.10** In a uniform gravitational field, the center of mass and center of gravity of an object coincide. Thus, the center of gravity of the triangle is located at $x = 6.67$ m, $y = 2.33$ m (see Example 9.14).

The coordinates of the three-object system are then:

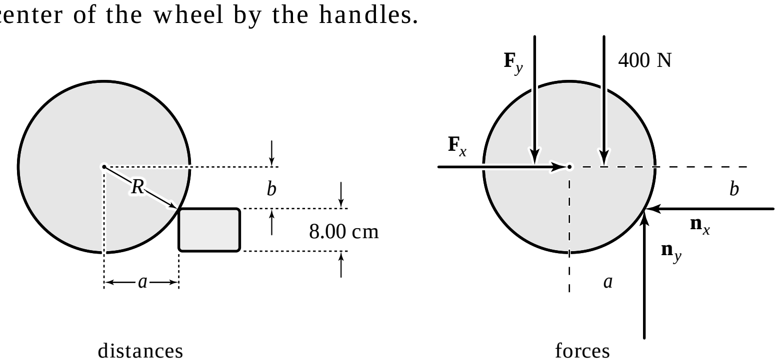
$$x_{CG} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(6.00\text{ kg})(5.50\text{ m}) + (3.00\text{ kg})(6.67\text{ m}) + (5.00\text{ kg})(-3.50\text{ m})}{(6.00 + 3.00 + 5.00)\text{ kg}}$$

$$x_{CG} = \frac{35.5\text{ kg}\cdot\text{m}}{14.0\text{ kg}} = \boxed{2.54\text{ m}} \quad \text{and}$$

$$y_{CG} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(6.00\text{ kg})(7.00\text{ m}) + (3.00\text{ kg})(2.33\text{ m}) + (5.00\text{ kg})(+3.50\text{ m})}{14.0\text{ kg}}$$

$$y_{CG} = \frac{69.5\text{ kg}\cdot\text{m}}{14.0\text{ kg}} = \boxed{4.75\text{ m}}$$

12.11 Call the required force F, with components $F_x = F\cos 15.0°$ and $F_y = -F\sin 15.0°$, transmitted to the center of the wheel by the handles.



Just as the wheel leaves the ground, the ground exerts no force on it.

$$\sum F_x = 0: \; F\cos 15.0° - n_x \qquad (1)$$

$$\sum F_y = 0: \; -F\sin 15.0° - 400\text{ N} + n_y = 0 \qquad (2)$$

Take torques about its contact point with the brick. The needed distances are seen to be:

$$b = R - 8.00\text{ cm} = (20.0 - 8.00)\text{ cm} = 12.0\text{ cm}$$

$$a = \sqrt{R^2 - b^2} = 16.0\text{ cm}$$

(a) $\sum \tau = 0: \; -F_x b + F_y a + (400\text{ N})a = 0$, or

$$F[-(12.0\text{ cm})\cos 15.0° + (16.0\text{ cm})\sin 15.0°] + (400\text{ N})(16.0\text{ cm}) = 0$$

so $$F = \frac{6400\text{ N}\cdot\text{cm}}{7.45\text{ cm}} = \boxed{859\text{ N}}$$

(b) Then, using Equations (1) and (2),

$n_x = (859\text{ N})\cos 15.0° = 830\text{ N}$ and

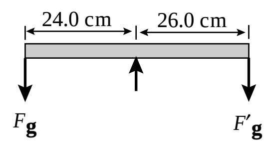
$n_y = 400\text{ N} + (859\text{ N})\sin 15.0° = 622\text{ N}$

$n = \sqrt{n_x^2 + n_y^2} = \boxed{1.04\text{ kN}}$

$\theta = \tan^{-1}\left(\dfrac{n_y}{n_x}\right) = \tan^{-1}(0.749) = \boxed{36.9° \text{ to the left and upward}}$

12.12 $F_g \rightarrow$ standard weight

$F'_g \rightarrow$ weight of goods sold



$F_g(0.240) = F'_g(0.260)$

$F_g = F'_g\left(\dfrac{13}{12}\right)$

$\left(\dfrac{F_g - F'_g}{F'_g}\right)100 = \left(\dfrac{13}{12} - 1\right) \times 100 = \boxed{8.33\%}$

12.13 (a) $\sum F_x = f - n_w = 0$

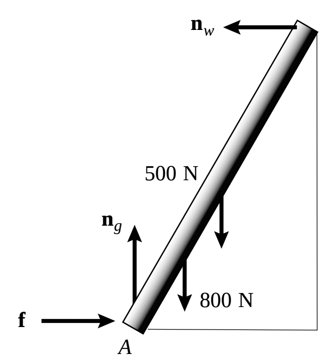
$\sum F_y = n_g - 800\text{ N} - 500\text{ N} = 0$

Taking torques about an axis at the foot of the ladder,

$(800\text{ N})(4.00\text{ m})\sin 30.0° + (500\text{ N})(7.50\text{ m})\sin 30.0° - n_w(15.0\text{ m})\cos 30.0° = 0$

Solving the torque equation,

$n_w = \dfrac{[(4.00\text{ m})(800\text{ N}) + (7.50\text{ m})(500\text{ N})]\tan 30.0°}{15.0\text{ m}} =$
268 N



Next substitute this value into the F_x equation to find

$f = n_w = \boxed{268\text{ N}}$ in the positive x direction

Solving the equation $\sum F_y = 0$,

$n_g = \boxed{1300\text{ N}}$ in the positive y direction

(b) In this case, the torque equation $\sum\tau_A = 0$ gives:

$$(9.00 \text{ m})(800 \text{ N}) \sin 30.0° + (7.50 \text{ m})(500 \text{ N}) \sin 30.0° - (15.0 \text{ m})(n_w) \sin 60.0° = 0$$

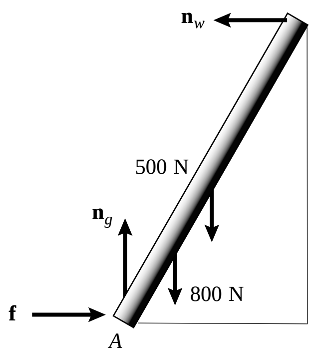
or $n_w = 421$ N

Since $f = n_w = 421$ N and $f = f_{max} = \mu n_g$, we find

$$\mu = \frac{f_{max}}{n_g} = \frac{421 \text{ N}}{1300 \text{ N}} = \boxed{0.324}$$

Goal Solution

G: Since the wall is frictionless, only the ground exerts an upward force on the ladder to oppose the combined weight of the ladder and firefighter, so $n_g = 1300$ N. Based on the angle of the ladder, $f < 1300$ N. The coefficient of friction is probably somewhere between 0 and 1.



O: Draw a free-body diagram, apply Newton's second law, and sum torques to find the unknown forces. Since this is a statics problem (no motion), both the net force and net torque are zero.

A: (a) $\sum F_x = f - n_w = 0$

$\sum F_y = n_g - 800 \text{ N} - 500 \text{ N} = 0$ so that $n_g = 1300$ N (upwards)

Taking torques about an axis at the foot of the ladder, $\sum\tau_A = 0$

$$-(800 \text{ N})(4.00 \text{ m}) \sin 30° - (500 \text{ N})(7.50 \text{ m}) \sin 30° + n_w(15.0 \text{ m}) \cos 30° = 0$$

Solving the torque equation for n_w,

$$n_w = \frac{[(4.00 \text{ m})(800 \text{ N}) + (7.50 \text{ m})(500 \text{ N})]}{15.0 \text{ m}} = 267.5 \text{ N}$$

Next substitute this value into the F_x equation to find

$f = n_w = 268$ N (**f** is directed toward the wall)

(b) When the firefighter is 9.00 m up the ladder, the torque equation $\sum \tau_A = 0$ gives

$$-(800\text{ N})(9.00\text{ m})\sin 30° - (500\text{ N})(7.50\text{ m})\sin 30° + n_w(15.0\text{ m})\sin 60° = 0$$

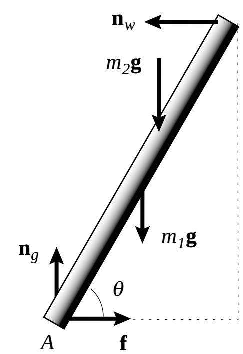
or $n_w = 421\text{ N}$

Since $f = n_w = 421$ N and $f = f_{max} = \mu_s n_g$,

$$\mu_s = \frac{f_{max}}{n_g} = \frac{421\text{ N}}{1300\text{ N}} = 0.324$$

L: The calculated answers seem reasonable since they agree with our predictions. This problem would be more realistic if the wall were not frictionless, in which case an additional vertical force would be added. This more complicated problem could be solved if we knew at least one of the coefficients of friction.

12.14 (a) $\sum F_x = f - n_w = 0$ (1)



$\sum F_y = n_g - m_1 g - m_2 g = 0$ (2)

$$\sum \tau_A = -m_1\left(\frac{L}{2}\right)\cos\theta - m_2 gx\cos\theta + n_w L\sin\theta = 0$$

From the torque equation,

$$n_w = \left[\frac{1}{2}m_1 g + \left(\frac{x}{L}\right)m_2 g\right]\cot\theta$$

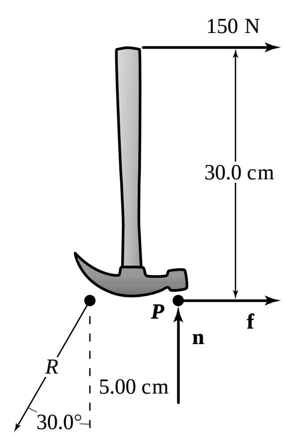
Then, from Equation (1): $f = n_w = \boxed{\left[\frac{1}{2}m_1 g + \left(\frac{x}{L}\right)m_2 g\right]\cot\theta}$

and from Equation (2): $n_g = \boxed{(m_1 + m_2)g}$

(b) If the ladder is on the verge of slipping when $x = d$, then

$$\mu = \frac{f\,|_{x=d}}{n_g} = \boxed{\frac{\left[\frac{1}{2}m_1 g + \left(\frac{d}{L}\right)m_2 g\right]\cot\theta}{(m_1 + m_2)g}}$$

12.15 (a) Taking moments about P,



$$(R\sin 30.0°)0 + (R\cos 30.0°)(5.00\text{ cm}) - (150\text{ N})(30.0\text{ cm}) = 0$$

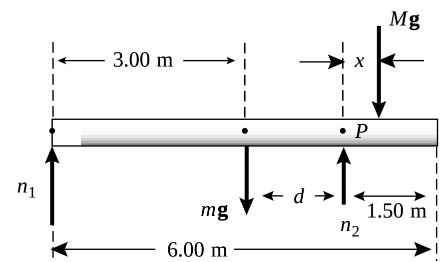
$$R = 1039.2\text{ N} = \boxed{1.04\text{ kN}}$$

(b) $f = R \sin 30.0° - 150 \text{ N} = 370 \text{ N}$

$n = R \cos 30.0° = 900 \text{ N}$

$\boxed{\mathbf{F}_{\text{surface}} = (370 \text{ N})\mathbf{i} + (900 \text{ N})\mathbf{j}}$

12.16 See the free-body diagram at the right.



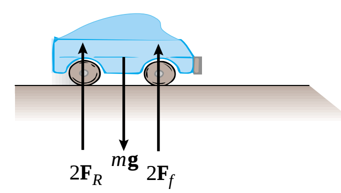
When the plank is on the verge of tipping about point P, the normal force n_1 goes to zero. Then, summing torques about point P gives

$$\Sigma\tau_p = -mgd + Mgx = 0 \quad \text{or} \quad x = \left(\frac{m}{M}\right) d$$

From the dimensions given on the free-body diagram, observe that $d = 1.50$ m. Thus, when the plank is about to tip,

$$x = \left(\frac{30.0 \text{ kg}}{70.0 \text{ kg}}\right)(1.50 \text{ m}) = \boxed{0.643 \text{ m}}$$

12.17 Torque about the front wheel is zero.



$$0 = (1.20 \text{ m})(mg) - (3.00 \text{ m})(2F_r)$$

Thus, the force at each rear wheel is $F_r = 0.200mg = \boxed{2.94 \text{ kN}}$

The force at each front wheel is then $F_f = \dfrac{mg - 2F_r}{2} = \boxed{4.41 \text{ kN}}$

Goal Solution

G: Since the center of mass lies in the front half of the car, there should be more force on the front wheels than the rear ones, and the sum of the wheel forces must equal the weight of the car.

O: Draw a free-body diagram, apply Newton's second law, and sum torques to find the unknown forces for this statics problem.

A: The car's weight is $F_g = mg = (1500 \text{ kg})(9.80 \text{ m/s}^2) = 14700 \text{ N}$

Call F the force of the ground on each of the front wheels and R the normal force on each of the rear wheels.

If we take torques around the front axle, the equations are as follows:

$\sum F_x = 0$ $\quad 0 = 0$

$\sum F_y = 0$ $\quad 2R - 14700\text{ N} + 2F = 0$

$\sum \tau = 0$ $\quad -2R(3.00\text{ m}) + (14700\text{ N})(1.20\text{ m}) + 2F(0) = 0$

The torque equation gives :

$$R = \frac{17\,640\text{ N}\cdot\text{m}}{6.00\text{ m}} = 2940\text{ N} = 2.94\text{ kN}$$

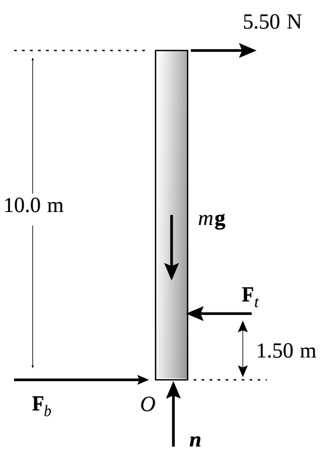
Then, from the second force equation,

$$2(2.94\text{ kN}) - 14.7\text{ kN} + 2F = 0$$

and $F = 4.41\text{ kN}$

L: As expected, the front wheels experience a greater force wheels (about 50% more) than the rear wheels. Since the frictional force between the tires and road is proportional to this normal force, it makes sense that most cars today are built with front wheel drive so that the wheels under power are the ones with more traction (friction).

***12.18** $\sum F_x = F_b - F_t + 5.50\text{ N} = 0$ (1)



$\sum F_y = n - mg = 0$

Summing torques about point O,

$$\sum \tau_O = F_t(1.50\text{ m}) - (5.50\text{ m})(10.0\text{ m}) = 0$$

which yields F_t = **36.7 N to the left**

Then, from Equation (1),

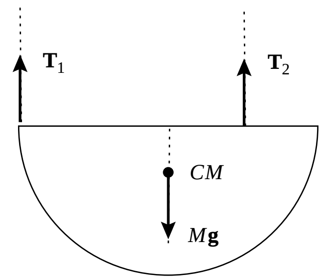
$$F_b = 36.7\text{ N} - 5.50\text{ N} = \boxed{31.2\text{ N to the right}}$$

12.19 (a) $T_e \sin 42.0° = 20.0\text{ N}$ $\quad \boxed{T_e = 29.9\text{ N}}$

(b) $T_e \cos 42.0° = T_m$ $\quad \boxed{T_m = 22.2\text{ N}}$

12.20 We call the tension in the cord at the left end of the sign, T_1, and the tension in the cord near the middle of the sign, T_2; and we choose our pivot point at the point where T_1 is attached.

$$\sum \tau_{\text{pivot}} = 0 = (-Mg)(0.500\text{ m}) + T_2(0.750\text{ m}) = 0,$$

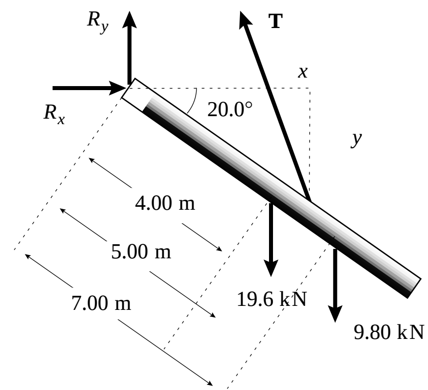


so, $T_2 = \boxed{\frac{2}{3}Mg}$

From $\sum F_y = 0$, $T_1 + T_2 - Mg = 0$

Substituting the expression for T_2 and solving, we find

$$T_1 = \boxed{\frac{1}{3}Mg}$$



12.21 Relative to the hinge end of the bridge, the cable is attached horizontally out a distance $x = (5.00\text{ m})\cos 20.0° = 4.70\text{ m}$ and vertically down a distance $y = (5.00\text{ m})\sin 20.0° = 1.71\text{ m}$. The cable then makes the following angle with the horizontal:

$$\theta = \tan^{-1}\left[\frac{(12.0 + 1.71)\text{ m}}{4.70\text{ m}}\right] = 71.1°$$

(a) Take torques about the hinge end of the bridge:

$$R_x(0) + R_y(0) - 19.6\text{ kN}(4.00\text{ m})\cos 20.0° - T\cos 71.1°(1.71\text{ m})$$

$$+\, T\sin 71.1°\,(4.70\text{ m}) - 9.80\text{ kN}(7.00\text{ m})\cos 20.0° = 0$$

which yields $T = \boxed{35.5\text{ kN}}$

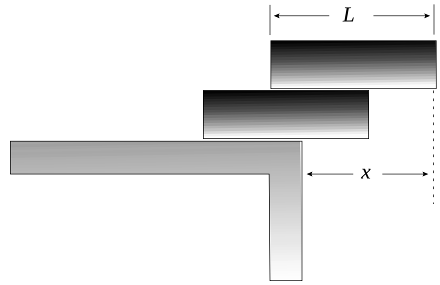
(b) $\sum F_x = 0 \Rightarrow R_x - T\cos 71.7° = 0$

or $R_x = (35.5\text{ kN})\cos 71.7° = \boxed{11.5\text{ kN (right)}}$

(c) $\sum F_y = 0 \Rightarrow R_y - 19.6\text{ kN} + T\sin 71.7° - 9.80\text{ kN} = 0$

Thus, $R_y = 29.4\text{ kN} - (35.5\text{ kN})\sin 71.7° = -4.19\text{ kN} = \boxed{4.19\text{ kN down}}$

12.22 $x = \boxed{\frac{3L}{4}}$



If the CM of the two bricks does not lie over the edge, then the bricks balance.

If the lower brick is placed $\frac{L}{4}$ over the edge, then the second brick may be placed so that its end protrudes $\frac{3L}{4}$ over the edge.

12.23 To find U, measure distances and forces from point A. Then, balancing torques,

$$(0.750)U = 29.4(2.25) \qquad \boxed{U = 88.2\text{ N}}$$

To find D, measure distances and forces from point B. Then, balancing torques,

$$(0.750)D = (1.50)(29.4) \qquad \boxed{D = 58.8\text{ N}}$$

Also, notice that $U = D + F_g$, so $\sum F_y = 0$

12.24 (a) stress $= F/A = F/\pi r^2$

$$F = (\text{stress})\pi(d/2)^2$$

$$F = (1.50 \times 10^8\text{ N/m}^2)\pi(2.50 \times 10^{-2}\text{ m}/2)^2$$

$$F = \boxed{73.6\text{ kN}}$$

(b) stress $= Y(\text{strain}) = Y\Delta L/L_i$

$$\Delta L = \frac{(\text{stress})L_i}{Y} = \frac{(1.50 \times 10^8\text{ N/m}^2)(0.250\text{ m})}{1.50 \times 10^{10}\text{ N/m}^2} = \boxed{2.50\text{ mm}}$$

12.25 $\frac{F}{A} = Y\frac{\Delta L}{L_i}$

$$\Delta L = \frac{FL_i}{AY} = \frac{(200)(9.80)(4.00)}{(0.200 \times 10^{-4})(8.00 \times 10^{10})} = \boxed{4.90\text{ mm}}$$

Goal Solution

G: Since metal wire does not stretch very much, the length will probably not change by more than 1% (<4 cm in this case) unless it is stretched beyond its elastic limit.

O: Apply the Young's Modulus strain equation to find the increase in length.

A: Young's Modulus is: $Y = \frac{F/A}{\Delta L/L_i}$

The load force is $F = (200\text{ kg})(9.80\text{ m/s}^2) = 1960\text{ N}$

so $\Delta L = \frac{FL_0}{AY} = \frac{(1960\text{ N})(4.00\text{ m})(1000\text{ mm/m})}{(0.200 \times 10^{-4}\text{ m}^2)(8.00 \times 10^{10}\text{ N/m}^2)} = 4.90\text{ mm}$

L: The wire only stretched about 0.1% of its length, so this seems like a reasonable result.

***12.26** Count the wires. If they are wrapped together so that all support nearly equal stress, the number should be

$$\frac{20.0\ \text{kN}}{0.200\ \text{kN}} = 100$$

Since cross-sectional area is proportional to diameter squared, the diameter of the cable will be

$$(1\ \text{mm})\sqrt{100}\ \boxed{\sim 1\ \text{cm}}$$

***12.27** From the defining equation for the shear modulus, we find Δx as

$$\Delta x = \frac{hf}{SA} = \frac{(5.00 \times 10^{-3}\ \text{m})(20.0\ \text{N})}{(3.0 \times 10^{6}\ \text{N/m}^2)(14.0 \times 10^{-4}\ \text{m}^2)} = 2.38 \times 10^{-5}\ \text{m}$$

or $\Delta x = \boxed{2.38 \times 10^{-2}\ \text{mm}}$

***12.28** The force acting on the hammer changes its momentum according to

$$mv_i + \bar{F}(\Delta t) = mv_f \quad \text{so} \quad \left|\bar{F}\right| = \frac{m\left|v_f - v_i\right|}{\Delta t}$$

Hence, $\left|\bar{F}\right| = \dfrac{30.0\ \text{kg}\left|-10.0\ \text{m/s} - 20.0\ \text{m/s}\right|}{0.110\ \text{s}} = 8.18 \times 10^{3}\ \text{N}$

By Newton's third law, this is also the magnitude of the average force exerted on the spike by the hammer during the blow. Thus, the stress in the spike is:

$$\text{stress} = \frac{F}{A} = \frac{8.18 \times 10^{3}\ \text{N}}{\pi(0.0230\ \text{m})^2/4} = 1.97 \times 10^{7}\ \text{N/m}^2$$

and the strain is: $\text{strain} = \dfrac{\text{stress}}{Y} = \dfrac{1.97 \times 10^{7}\ \text{N/m}^2}{20.0 \times 10^{10}\ \text{N/m}^2} = \boxed{9.85 \times 10^{-5}}$

12.29 In this problem, $F = mg = 10.0(9.80) = 98.0\ \text{N}$, $A = \pi d^2/4$,

and the maximum stress $= \dfrac{F}{A} = 1.50 \times 10^{8}\ \text{N/m}^2$

$$A = \frac{\pi d^2}{4} = \frac{F}{\text{Stress}} = \frac{98.0\ \text{N}}{1.50 \times 10^{8}\ \text{N/m}^2} = 6.53 \times 10^{-7}\ \text{m}^2$$

$$d^2 = \frac{4(6.53 \times 10^{-7}\ \text{m}^2)}{\pi}$$

$$d = 9.12 \times 10^{-4}\ \text{m} = \boxed{0.912\ \text{mm}}$$

12.30 Let the 3.00 kg mass be mass #1, with the 5.00 kg mass, mass # 2. Applying Newton's second law to each mass gives:

$$m_1 a = T - m_1 g \quad (1) \qquad \text{and} \qquad m_2 a = m_2 g - T \quad (2)$$

where T is the tension in the wire.

Solving equation (1) for the acceleration gives: $a = \dfrac{T}{m_1} - g$,

and substituting this into equation (2) yields: $\dfrac{m_2}{m_1} T - m_2 g = m_2 g - T$

Solving for the tension T gives

$$T = \frac{2m_1 m_2 g}{m_2 + m_1} = \frac{2(3.00\text{ kg})(5.00\text{ kg})(9.80\text{ m/s}^2)}{8.00\text{ kg}} = 36.8\text{ N}$$

From the definition of Young's modulus, $Y = \dfrac{FL_i}{A(\Delta L)}$, the elongation of the wire is:

$$\Delta L = \frac{TL_i}{YA} = \frac{(36.8\text{ N})(2.00\text{ m})}{(2.00 \times 10^{11}\text{ N/m}^2)\,\pi\,(2.00 \times 10^{-3}\text{ m})^2} = \boxed{0.0293\text{ mm}}$$

12.31 Assume that $m_2 > m_1$. Then, application of Newton’s second law to each mass yields the following equations of motion:

$$T - m_1 g = m_1 a \quad (1) \qquad \text{and} \qquad m_2 g - T = m_2 a \quad (2)$$

Solving Equation (1) for the acceleration gives $a = \dfrac{T}{m_1} - g$

and substitution into Equation (2) yields $m_2 g - T = \left(\dfrac{m_2}{m_1}\right) T - m_2 g$

The tension in the wire is then: $T = \dfrac{2m_2 g}{(m_1 + m_2)/m_1} = \dfrac{2m_1 m_2 g}{m_1 + m_2}$

From the definition of Young’s modulus, $Y = \dfrac{FL_i}{A(\Delta L)}$, the elongation of the wire is found to be:

$$\Delta L = \frac{TL_i}{AY} = \frac{[2m_1 m_2 g/(m_1 + m_2)]L_i}{(\pi d^2/4)Y} = \boxed{\frac{8m_1 m_2 g L_i}{\pi d^2 Y(m_1 + m_2)}}$$

12.32 At the surface 1030 kg of water fills 1.00 m^3. A kilometer down its volume has shrunk by ΔV in

$$\Delta V = \frac{-(\Delta P)V_i}{B} = \frac{-(10^7\ \text{N/m}^2)(1.00\ \text{m}^3)}{0.210 \times 10^{10}\ \text{N/m}^2} = -4.76 \times 10^{-3}\ \text{m}^3$$

so the new volume is $V = 1.00\ \text{m}^3 - 4.76 \times 10^{-3}\ \text{m}^3 = 0.99524\ \text{m}^3$

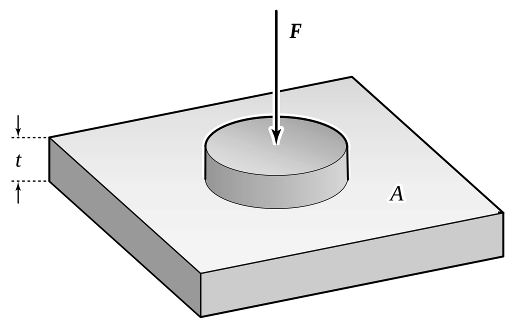
$$\therefore \text{ its density is } \rho = \frac{m}{V} = \frac{1030\ \text{kg}}{0.99524\ \text{m}^3} = \boxed{1.035 \times 10^3\ \text{kg/m}^3}$$

12.33 (a) $F = (A)(\text{stress}) = \pi(5.00 \times 10^{-3}\ \text{m})^2(4.00 \times 10^8\ \text{N/m}^2) = \boxed{3.14 \times 10^4\ \text{N}}$

(b) The area over which the shear occurs is equal to the circumference of the hole times its thickness. Thus,

$$A = (2\pi r)t = 2\pi(5.00 \times 10^{-3}\ \text{m})(5.00 \times 10^{-3}\ \text{m}) = 1.57 \times 10^{-4}\ \text{m}^2$$

So, $F = (A)\text{Stress} = (1.57 \times 10^{-4}\ \text{m}^2)(4.00 \times 10^8\ \text{N/m}^2) = \boxed{6.28 \times 10^4\ \text{N}}$



12.34 (a) Using $Y = \dfrac{FL_i}{A(\Delta L)}$, we get $A = \dfrac{FL_i}{Y(\Delta L)} = \pi(d/2)^2$

So, $$d = \sqrt{\frac{4mgL_i}{\pi Y(\Delta L)}} = \sqrt{\frac{4(380\ \text{kg})(9.80\ \text{m/s}^2)(18.0\ \text{m})}{\pi(2.00 \times 10^{11}\ \text{N/m}^2)(9.00 \times 10^{-3}\ \text{m})}} = \boxed{6.89\ \text{mm}}$$

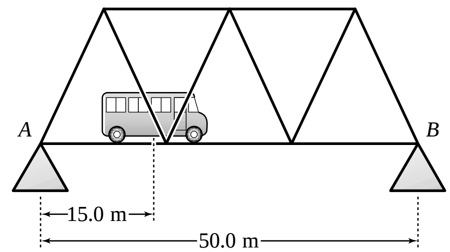
(b) $A = 3.72 \times 10^{-5}\ \text{m}^2$ $\quad F/A = 1.00 \times 10^8\ \text{N/m}^2$ $\boxed{\text{No}}$

12.35 $$\Delta P = -B\left(\frac{\Delta V}{V_i}\right) = -\left(2.00 \times 10^9\ \frac{\text{N}}{\text{m}^2}\right)(-0.090) = \boxed{1.80 \times 10^8\ \text{N/m}^2} \approx 1800\ \text{atm}$$

12.36 Using $Y = \dfrac{FL_i}{A(\Delta L)}$ with $A = \pi(d/2)^2$ and $F = mg$, we get

$$Y = \frac{4mgL_i}{\pi d^2(\Delta L)} = \frac{4(90.0\ \text{kg})(9.80\ \text{m/s}^2)(50.0\ \text{m})}{\pi(0.0100\ \text{m})^2(1.60\ \text{m})} = \boxed{3.51 \times 10^8\ \text{N/m}^2}$$

12.37 Let n_A and n_B be the normal forces at the points of support.



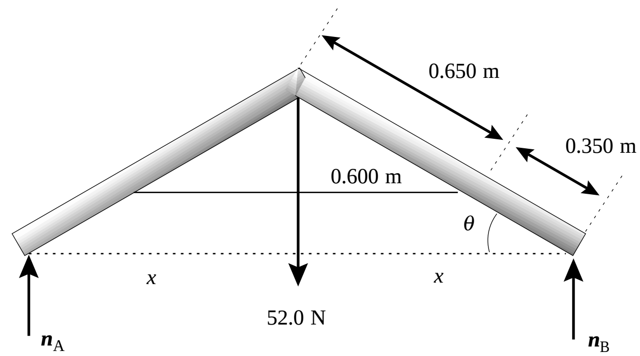
Choosing the origin at point A with $\sum F_y = 0$ and $\sum \tau = 0$, we find:

$$n_A + n_B - (8.00 \times 10^4)g - (3.00 \times 10^4)g = 0 \quad \text{and}$$

$$-(3.00 \times 10^4)(g)15.0 - (8.00 \times 10^4)(g)25.0 + n_B(50.0) = 0$$

The equations combine to give $n_A =$ $\boxed{5.98 \times 10^5\ \text{N}}$ and $n_B = \boxed{4.80 \times 10^5\ \text{N}}$

12.38 Using similar triangles in the first figure at the right, the horizontal extent of each bar is found as

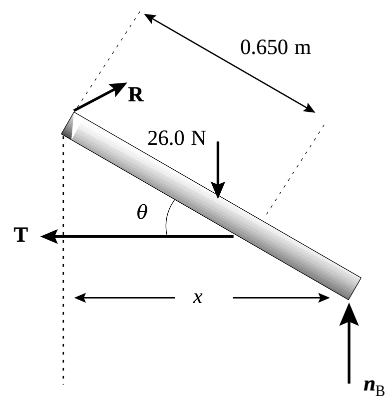


$$\frac{x}{(0.650 + 0.350)\ \text{m}} = \frac{0.600\ \text{m}}{0.650\ \text{m}}$$

or $x = 0.923$ m. The angle each bar makes with the horizontal is

$$\theta = \cos^{-1}\left(\frac{x}{1.00\ \text{m}}\right) = \cos^{-1}(0.923)$$

or $\theta = 22.6°$



choose the whole frame as object and take torques about point A, its left contact with the ground:

$$-(52.0\ \text{N})x + n_B(2x) = 0$$

giving $n_B = 26.0$ N

Isolate the right-side bar and take torques about its upper end:

$$R(0) - (26.0\ \text{N})[(0.500\ \text{m})\cos\theta] - (T\sin\theta)(0.650\ \text{m}) + n_B x = 0$$

so $$T = \frac{(26.0\ \text{N})(0.923\ \text{m}) - (26.0\ \text{N})(0.500\ \text{m})\cos 22.6°}{(0.650\ \text{m})\sin 22.6°} = \boxed{48.0\ \text{N}}$$

***12.39** When the concrete has cured and the pre-stressing tension has been released, the rod presses in on the concrete and with equal force, T_2, the concrete produces tension in the rod.

(a) In the concrete: stress $= 8.00 \times 10^6\ \text{N/m}^2 = Y \cdot (\text{strain}) = Y(\Delta L / L_i)$

Thus, $$\Delta L = \frac{(\text{stress})L_i}{Y} = \frac{(8.00 \times 10^6\ \text{N/m}^2)(1.50\ \text{m})}{30.0 \times 10^9\ \text{N/m}^2}$$

or $\Delta L = 4.00 \times 10^{-4}\ \text{m} = \boxed{0.400\ \text{mm}}$

(b) In the concrete: stress $= \dfrac{T_2}{A_c} = 8.00 \times 10^6 \text{ N/m}^2$, so

$$T_2 = (8.00 \times 10^6 \text{ N/m}^2)(50.0 \times 10^{-4} \text{ m}^2) = \boxed{40.0 \text{ kN}}$$

(c) For the rod: $\dfrac{T_2}{A_R} = \left(\dfrac{\Delta L}{L_i}\right) \Upsilon_{\text{steel}}$ so $\Delta L = \dfrac{T_2 L_i}{A_R \Upsilon_{\text{steel}}}$

$$\Delta L = \frac{(4.00 \times 10^4 \text{ N})(1.50 \text{ m})}{(1.50 \times 10^{-4} \text{ m}^2)(20.0 \times 10^{10} \text{ N/m}^2)} = 2.00 \times 10^{-3} \text{ m} = \boxed{2.00 \text{ mm}}$$

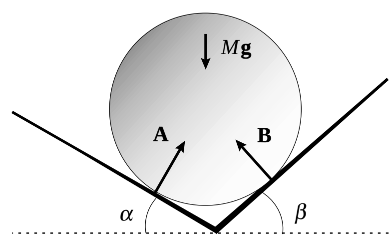
(d) The rod in the finished concrete is 2.00 mm longer than its unstretched length. To remove stress from the concrete, one must stretch the rod 0.400 mm farther, by a total of $\boxed{2.40 \text{ mm}}$.

(e) For the stretched rod around which the concrete is poured:

$$\frac{T_1}{A_R} = \left(\frac{\Delta L_{\text{total}}}{L_i}\right) \Upsilon_{\text{steel}} \quad \text{or} \quad T_1 = \left(\frac{\Delta L_{\text{total}}}{L_i}\right) A_R \Upsilon_{\text{steel}}$$

$$T_1 = \left(\frac{2.40 \times 10^{-3} \text{ m}}{1.50 \text{ m}}\right)(1.50 \times 10^{-4} \text{ m}^2)(20.0 \times 10^{10} \text{ N/m}^2) = \boxed{48.0 \text{ kN}}$$

12.40 Call the normal forces A and B. They make angles α and β with the vertical.



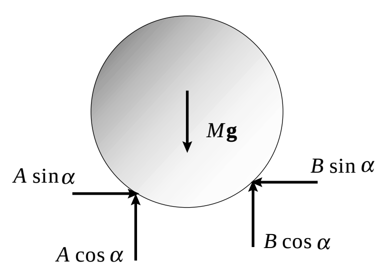
$$\sum F_x = 0: \ A \sin \alpha - B \sin \beta = 0$$

$$\sum F_y = 0: \ A \cos \alpha - Mg + B \cos \beta = 0$$

Substitute $B = A \sin \alpha / \sin \beta$

$$A \cos \alpha + A \cos \beta \sin \alpha / \sin \beta = Mg$$

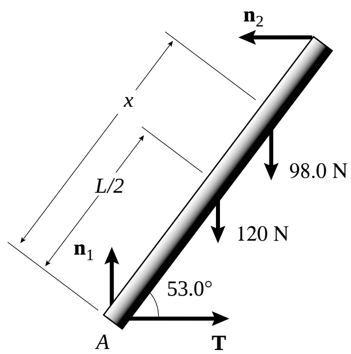
$$A(\cos \alpha \sin \beta + \sin \alpha \cos \beta) = Mg \sin \beta$$



$$A = \boxed{Mg \frac{\sin \beta}{\sin (\alpha + \beta)}}$$

$$B = \boxed{Mg \frac{\sin \alpha}{\sin (\alpha + \beta)}}$$

12.41 (a) See figure.



(b) Using $\sum F_x = \sum F_y = \sum \tau = 0$, we have (with A the bottom of the ladder):

$$\sum F_x = T - n_2 = 0$$

$$\sum F_y = n_2 - 218 \text{ N} = 0$$

$$\sum \tau_A = 98.0 \cos 53.0° + 120\left(\frac{L}{2}\right) \cos 53.0° - n_2 L \sin 53.0° = 0$$

where x is the distance of the monkey from the bottom of the ladder. When $x = L/3$, the above equation gives

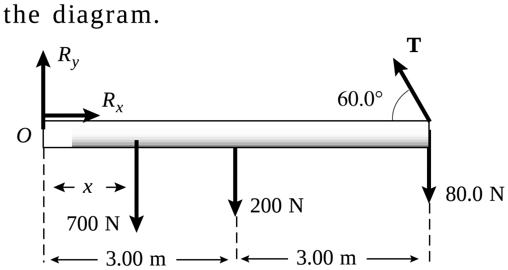
$$T = \frac{(18.7 + 36.1)}{0.800} = \boxed{69.8 \text{ N}}$$

(c) The rope breaks when $T = 110 \text{ N} = n_2$

$$\sum \tau_A = 10.0(9.80)x \cos 53.0° + 120(L/2) \cos 53.0° - 110L \sin 53.0° = 0$$

$$x = \frac{100L \sin 53.0° - 60.0L (\cos 53.0°)}{10.0(9.80) \cos 53.0°} = \boxed{0.877L}$$

12.42 (a) See the diagram.



(b) If $x = 1.00$ m, then

$$\sum \tau_O = (-700 \text{ N})(1.00 \text{ m}) - (200 \text{ N})(3.00 \text{ m}) - (80.0 \text{ N})(6.00 \text{ m})$$

$$+ (T \sin 60.0°)(6.00 \text{ m}) = 0$$

Solving for the tension gives: $T = \boxed{343\text{ N}}$

From $\sum F_x = 0$, $R_x = T\cos 60.0° = \boxed{171\text{ N}}$

From $\sum F_y = 0$, $R_y = 980\text{ N} - T\sin 60.0° = \boxed{683\text{ N}}$

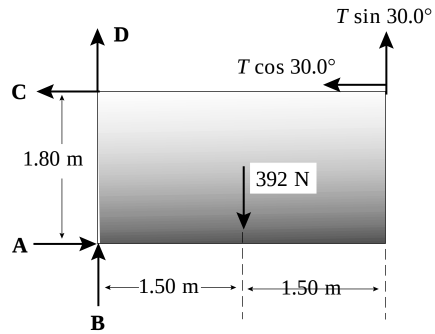
(c) If $T = 900$ N:

$$\sum \tau_O = (-700\text{ N})x - (200\text{ N})(3.00\text{ m}) - (80.0\text{ N})(6.00\text{ m})$$

$$+ [(900\text{ N})\sin 60.0°](6.00\text{ m}) = 0$$

Solving for x gives: $x = \boxed{5.13\text{ m}}$

12.43 (a) Sum the torques about top hinge:



$\sum \tau = 0$:

$$C(0) + D(0) + 200\text{ N}\cos 30.0°\,(0)$$

$$+ 200\text{ N}\sin 30.0°(3.00\text{ m})$$

$$- 392\text{ N}(1.50\text{ m}) + A(1.80\text{ m})$$

$$+ B(0) = 0$$

Giving $A = \boxed{160\text{ N (right)}}$

(b) $\sum F_x = 0$:

$$-C - 200\text{ N}\cos 30.0° + A = 0$$

$$C = 160\text{ N} - 173\text{ N} = -13.2\text{ N}$$

In our diagram, this means $\boxed{13.2\text{ N to the right}}$

(c) $\sum F_y = 0$: $+B + D - 392\text{ N} + 200\text{ N}\sin 30.0° = 0$

$B + D = 392\text{ N} - 100\text{ N} = \boxed{292\text{ N(up)}}$

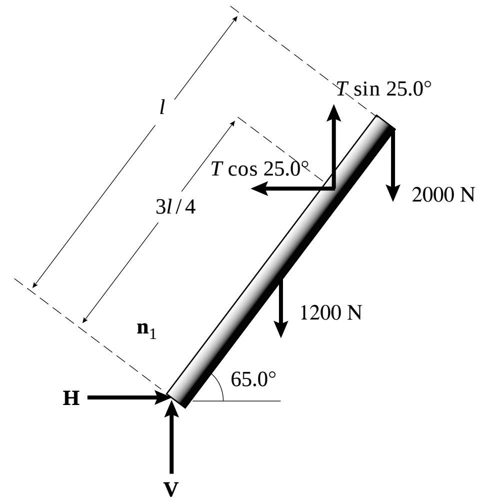
(d) Given $C = 0$: Take torques about bottom hinge to obtain

$$A(0) + B(0) + 0(1.80\text{ m}) + D(0) - 392\text{ N}(1.50\text{ m})$$

$$+ T\sin 30.0°(3.00\text{ m}) + T\cos 30.0°(1.80\text{ m}) = 0$$

so $$T = \frac{588\text{ N}\cdot\text{m}}{(1.50\text{ m} + 1.56\text{ m})} = \boxed{192\text{ N}}$$

12.44 $\sum \tau_{\text{point } 0} = 0$ gives



$$(T\cos 25.0°)\left(\frac{3l}{4}\sin 65.0°\right) + (T\sin 25.0°)\left(\frac{3l}{4}\cos 65.0°\right)$$

$$= (2000\text{ N})(l\cos 65.0°) + (1200\text{ N})\left(\frac{l}{2}\cos 65.0°\right)$$

From which, $T = 1465$ N = $\boxed{1.46\text{ kN}}$

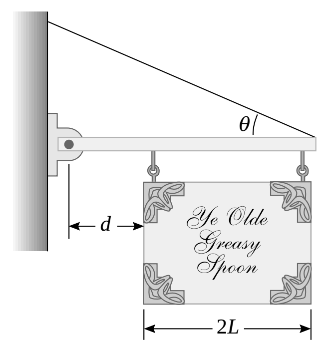
From $\sum F_x = 0$,

$H = T\cos 25.0° = 1328$ N (toward right) = $\boxed{1.33\text{ kN}}$

From $\sum F_y = 0$,

$V = 3200\text{ N} - T\sin 25.0° = 2581$ N (upward) = $\boxed{2.58\text{ kN}}$

12.45 Using $\sum F_x = \sum F_y = \sum \tau = 0$, choosing the origin at the left end of the beam, we have (neglecting the weight of the beam)



$\sum F_x = R_x - T\cos\theta = 0,$

$\sum F_y = R_y + T\sin\theta - F_g = 0,$ and

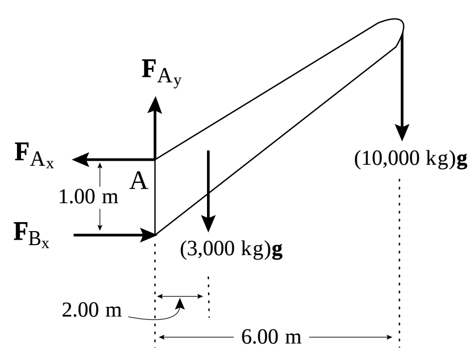
$\sum \tau = -F_g(L + d) + T\sin\theta\,(2L + d) = 0$

Solving these equations, we find:

(a) $T = \boxed{\dfrac{F_g(L + d)}{\sin\theta(2L + d)}}$ and

(b) $R_x = \boxed{\dfrac{F_g(L + d)\cot\theta}{2L + d}}$ $R_y = \boxed{\dfrac{F_g L}{2L + d}}$

12.46 At point B since the support is *smooth* the reaction force is in the x direction. If we choose point A as the origin, then we have



$\sum F_x = F_{Bx} - F_{Ax} = 0$

$F_{Ay} - (3000 + 10000)g = 0$

and $\sum \tau = -(3000g)(2.00) - (10000g)(6.00) + F_{Bx}(1.00) = 0$

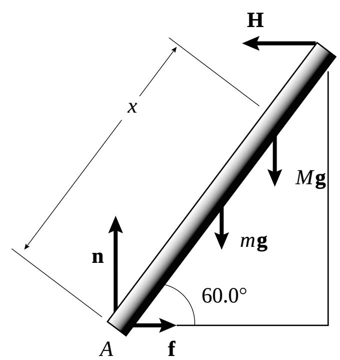
These equations combine to give

$$F_{Ax} = F_{Bx} = \boxed{6.47 \times 10^5 \text{ N}}$$

and $F_{By} = \boxed{0}$

$$\boxed{F_{Ay} = 1.27 \times 10^5 \text{ N}}$$

12.47 $n = (M + m)g \quad H = f$



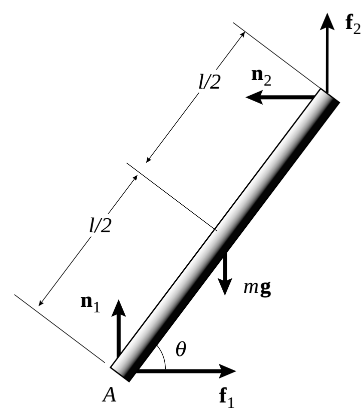
$$H_{\max} = f_{\max} = \mu_s(m + M)g$$

$$\sum\tau_A = 0 = \frac{mgL}{2}\cos 60.0° + Mgx\cos 60.0° - HL\sin 60.0°$$

$$\frac{x}{L} = \frac{H\tan 60.0°}{Mg} - \frac{m}{2M} = \frac{\mu_s(m + M)\tan 60.0°}{M} - \frac{m}{2M}$$

$$= \frac{3}{2}\mu_s\tan 60.0° - \frac{1}{4} = \boxed{0.789}$$

12.48 Since the ladder is about to slip, $f = (f_s)_{\max} = \mu_s n$ at each contact point. Because the ladder is still (barely) in equilibrium: $\sum F_x = 0$, which gives



$$f_1 - n_2 = 0 \quad \text{or} \quad \mu_s n_1 = n_2$$

giving $\quad n_1 = \dfrac{n_2}{\mu_s}$

Since $\sum F_y = 0$, use $n_1 - mg + f_2$ to eliminate n_1,

$$n_2 = \frac{\mu_s mg}{1 + \mu_s^2} \quad (1)$$

$$\sum\tau_{\text{lower end}} = 0 \quad \text{gives} \quad -mg\frac{L}{2}\cos\theta + n_2 L\sin\theta + f_2 L\cos\theta = 0$$

which can be written as $-\dfrac{mg}{2} + n_2\tan\theta + \mu_s n_2 = 0$, or

$$mg = 2n_2(\tan\theta + \mu_s) \qquad (2)$$

Substituting equation (1) into equation (2) gives

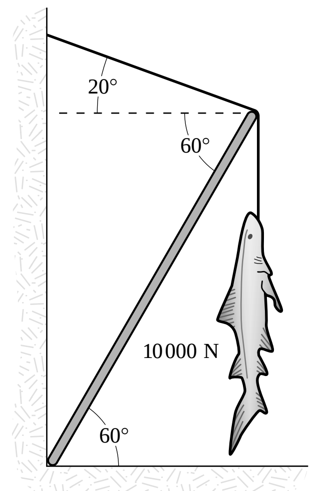
$$mg = \frac{2\mu_s mg(\tan\,\theta + \mu_s)}{1 + \mu_s^2} \text{ which reduces to}$$

$$1 + \mu_s^2 = 2\mu_s \tan\,\theta + 2\mu_s^2 \quad \text{or} \quad \mu_s^2 + (2\tan\,\theta)\mu_s - 1 = 0$$

With $\theta = 60.0°$, this becomes $\mu_s^2 + 3.646\mu_s - 1 = 0$,

which has one positive solution: $\mu_s = \boxed{0.268}$

12.49 Summing torques around the base of the rod,



$$\sum\tau = -(4.00\text{ m})(10000\text{ N})\cos 60.0° + T(4.00\text{ m})\sin 80.0° = 0$$

$$T = \frac{(10000\text{ N})\cos 60.0°}{\sin 80.0°} = \boxed{5.08 \times 10^3\text{ N}}$$

Since $F_H - T\cos 20.0° = 0$, $F_H = \boxed{4.77\text{ kN}}$

$$F_V + T\sin 20.0° - 10.0\text{ kN} = 0, \quad F_V = \boxed{8.26\text{ kN}}$$

Goal Solution

G: Since the rod helps support the weight of the shark by exerting a vertical force, the tension in the upper portion of the cable must be less than 10 000 N. Likewise, the vertical and horizontal forces on the base of the rod should also be less than 10 kN.

O: This is another statics problem where the sum of the forces and torques must be zero. To find the unknown forces, draw a free-body diagram, apply Newton's second law, and sum torques.

A: From the free-body diagram, the angle **T** makes with the rod is

$$\theta = 60.0° + 20.0° = 80.0°$$

and the perpendicular component of **T** is $T\sin 80.0°$.

Summing torques around the base of the rod,

$$\sum\tau = 0: \quad -(4.00\text{ m})(10000\text{ N})\cos 60° + T(4.00\text{ m})\sin 80° = 0$$

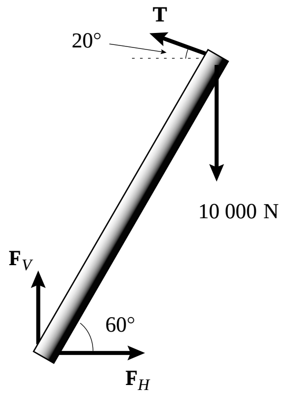
$$T = \frac{(10000\text{ N})\cos 60.0°}{\sin 80.0°} = 5.08 \times 10^3\text{ N}$$

$\sum F_x = 0$: $F_H - T\cos 20.0° = 0$

$F_H = T\cos 20.0° = 4.77 \times 10^3$ N

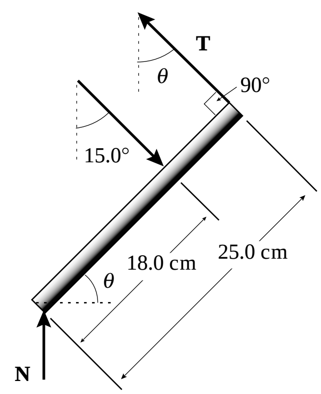
$\sum F_y = 0$: $F_V + T\sin 20.0° - 10000 \text{ N} = 0$

and $F_V = (10000 \text{ N}) - T\sin 20.0° = 8.26 \times 10^3$ N



L: The forces calculated are indeed less than 10 kN as predicted. That shark sure is a big catch; it weighs about a ton!

12.50 Choosing the origin at **R**,



(1) $\sum F_x = +R\sin 15.0° - T\sin\theta = 0$

(2) $\sum F_y = 700 - R\cos 15.0° + T\cos\theta = 0$

(3) $\sum \tau = -700\cos\theta(0.180) + T(0.0700) = 0$

Solve the equations for θ

from (3), $T = 1800\cos\theta$ from (1), $R = \dfrac{1800\sin\theta\cos\theta}{\sin 15.0°}$

Then (2) gives $700 - \dfrac{1800\sin\theta\cos\theta\cos 15.0°}{\sin 15.0°} + 1800\cos^2\theta = 0$

or $\cos^2\theta + 0.3889 - 3.732\sin\theta\cos\theta = 0$

Squaring, $\cos^4\theta - 0.8809\cos^2\theta + 0.01013 = 0$

Let $u = \cos^2\theta$ then using the quadratic equation,

$u = 0.01165$ or 0.8693

Only the second root is physically possible,

$\therefore \theta = \cos^{-1}\sqrt{0.8693} = \boxed{21.2°}$

$\therefore T = \boxed{1.68 \times 10^3 \text{ N}}$ and $R = \boxed{2.34 \times 10^3 \text{ N}}$