CHAPTER 17 | THE PRINCIPLE OF LINEAR SUPERPOSITION AND INTERFERENCE PHENOMENA

Homework Solutions: #'s 1, 5, 7, 10, 16, 23, 34. Study for the Test.

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1. ***REASONING AND SOLUTION*** In a time of t = 1 s, the pulse on the left has moved to the right a distance of 1 cm, while the pulse on the right has moved to the left a distance of 1 cm. Adding the shapes of these two pulses when t = 1 s reveals that the height of the resultant pulse is

   a. $\boxed{2\text{ cm}}$ at x = 3 cm.

   b. $\boxed{1\text{ cm}}$ at x = 4 cm.

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5. ***REASONING*** The tones from the two speakers will produce destructive interference with the smallest frequency when the path length difference at C is one-half of a wavelength. From Figure 17.7, we see that the path length difference is $\Delta s = s_{\text{AC}} - s_{\text{BC}}$. From Example 1, we know that $s_{\text{AC}} = 4.00\text{ m}$, and from Figure 17.7, $s_{\text{BC}} = 2.40\text{ m}$. Therefore, the path length difference is $\Delta s = 4.00\text{ m} - 2.40\text{ m} = 1.60\text{ m}$.

   SOLUTION Thus, destructive interference will occur when

$$\frac{\lambda}{2} = 1.60\text{ m} \qquad \text{or} \qquad \lambda = 3.20\text{ m}$$

   This corresponds to a frequency of

$$f = \frac{v}{\lambda} = \frac{343\text{ m/s}}{3.20\text{ m}} = \boxed{107\text{ Hz}}$$

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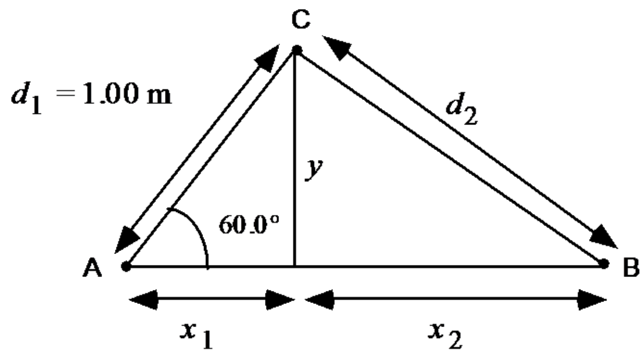
7. ***REASONING*** The geometry of the positions of the loudspeakers and the listener is shown in the following drawing.



The listener at C will hear either a loud sound or no sound, depending upon whether the interference occurring at C is constructive or destructive. If the listener hears no sound, destructive interference occurs, so

$$d_2 - d_1 = \frac{n\lambda}{2} \qquad n = 1, 3, 5, \ldots \tag{1}$$

SOLUTION Since $v = \lambda f$, according to Equation 16.1, the wavelength of the tone is

$$\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{68.6 \text{ Hz}} = 5.00 \text{ m}$$

Speaker B will be closest to Speaker A when $n = 1$ in Equation (1) above, so

$$d_2 = \frac{n\lambda}{2} + d_1 = \frac{5.00 \text{ m}}{2} + 1.00 \text{ m} = 3.50 \text{ m}$$

From the figure above we have that,

$$x_1 = (1.00 \text{ m}) \cos 60.0° = 0.500 \text{ m}$$

$$y = (1.00 \text{ m}) \sin 60.0° = 0.866 \text{ m}$$

Then

$$x_2^2 + y^2 = d_2^2 = (3.50 \text{ m})^2 \qquad \text{or} \qquad x_2 = \sqrt{(3.50 \text{ m})^2 - (0.866 \text{ m})^2} = 3.39 \text{ m}$$

Therefore, the closest that speaker A can be to speaker B so that the listener hears no sound is $x_1 + x_2 = 0.500 \text{ m} + 3.39 \text{ m} = \boxed{3.89 \text{ m}}$.

10. ***REASONING*** For a rectangular opening ("single slit") such as a doorway, the diffraction angle θ at which the first minimum in the sound intensity occurs is given by $\sin\theta = \dfrac{\lambda}{D}$ (Equation 17.1), where λ is the wavelength of the sound and D is the width of the opening. This relation can be used to find the angle provided we realize that the wavelength λ is related to the speed v of sound and the frequency f by $\lambda = v/f$ (Equation 16.1).

SOLUTION
a. Substituting $\lambda = v/f$ into Equation 17.1 and using D = 0.700 m (only one door is open) gives

$$\sin\theta = \frac{\lambda}{D} = \frac{v}{f\,D} = \frac{343\text{ m/s}}{(607\text{ Hz})(0.700\text{ m})} = 0.807 \qquad \theta = \sin^{-1}(0.807) = \boxed{53.8^\circ}$$

b. When both doors are open, D = 2 × 0.700 m and the diffraction angle is

$$\sin\theta = \frac{\lambda}{D} = \frac{v}{f\,D} = \frac{343\text{ m/s}}{(607\text{ Hz})(2\times 0.700\text{ m})} = 0.404 \qquad \theta = \sin^{-1}(0.404) = \boxed{23.8^\circ}$$

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16. ***REASONING*** When two frequencies are sounded simultaneously, the beat frequency produced is the difference between the two. Thus, knowing the beat frequency between the tuning fork and one flute tone tells us only the difference between the known frequency and the tuning-fork frequency. It does not tell us whether the tuning-fork frequency is greater or smaller than the known frequency. However, two different beat frequencies and two flute frequencies are given. Consideration of both beat frequencies will enable us to find the tuning-fork frequency.

SOLUTION The fact that a 1-Hz beat frequency is heard when the tuning fork is sounded along with the 262-Hz tone implies that the tuning-fork frequency is either 263 Hz or 261 Hz. We can eliminate one of these values by considering the fact that a 3-Hz beat frequency is heard when the tuning fork is sounded along with the 266-Hz tone. This implies that the tuning-fork frequency is either 269 Hz or 263 Hz. Thus, the tuning-fork frequency must be $\boxed{263\text{ Hz}}$.

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23. ***REASONING*** For standing waves on a string that is clamped at both ends, Equations 17.3 and 16.2 indicate that the standing wave frequencies are

$$f_n = n\left(\frac{v}{2L}\right) \qquad \text{where} \qquad v = \sqrt{\frac{F}{m/L}}$$

Combining these two expressions, we have, with $n = 1$ for the fundamental frequency,

$$f_1 = \frac{1}{2L}\sqrt{\frac{F}{m/L}}$$

This expression can be used to find the ratio of the two fundamental frequencies.

SOLUTION The ratio of the two fundamental frequencies is

$$\frac{f_{\text{old}}}{f_{\text{new}}} = \frac{\frac{1}{2L}\sqrt{\frac{F_{\text{old}}}{m/L}}}{\frac{1}{2L}\sqrt{\frac{F_{\text{new}}}{m/L}}} = \sqrt{\frac{F_{\text{old}}}{F_{\text{new}}}}$$

Since $F_{\text{new}} = 4F_{\text{old}}$, we have

$$f_{\text{new}} = f_{\text{old}}\sqrt{\frac{F_{\text{new}}}{F_{\text{old}}}} = f_{\text{old}}\sqrt{\frac{4\,F_{\text{old}}}{F_{\text{old}}}} = f_{\text{old}}\sqrt{4} = (55.0\text{ Hz})\,(2) = \boxed{1.10\times10^2\text{ Hz}}$$

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34. ***REASONING*** The frequency of a pipe open at both ends is given by Equation 17.4 as $f_{\text{n}} = n\left(\frac{v}{2L}\right)$, where n is an integer specifying the harmonic number, v is the speed of sound, and L is the length of the pipe. This relation can be used to find L, since all the other variables are known.

SOLUTION Solving the equation above for L, and recognizing that n = 3 for the third harmonic, we have

$$L = n\left(\frac{v}{2\,f_{\text{n}}}\right) = 3\left[\frac{343\text{ m/s}}{2(262\text{ Hz})}\right] = \boxed{1.96\text{ m}}$$
