Physics

Simply Put

Volume 2: Electricity and Magnetism

with Calculus

by

Dr. Ronald C. Persin

Lnk2Lrn.com

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Founder of Lnk2Lrn.com

"From Newtonian mechanics,

Through Quantum Theory,

Without Physics,

Life would be dreary."RCP

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Abstract

This book is a compilation of Dr. Persin's website notes that were originally published on his website, Lnk2Lrn.com. According to his many physics students over the years, these notes were especially helpful in understanding important concepts in calculus-based physics.

Topics include separate chapters on electric fields, Gauss' Law, electric potential, capacitance, current and resistance, DC circuits, magnetic effects, magnetic fields, Faraday's Law, and inductance.

Sample problems solved in step-by-step manner appear at the end of each chapter.

It is the intent of the author that this book will help students to be prepared for class on a daily basis with all the knowledge of the day before.

Chapter 1 - Electric Fields.

INTRODUCTION: In ancient Greece, amber became widely valued around 1600 BC. Greeks were fascinated by it. The ancient Greek word for amber is "**elektron**", meaning - originating from the Sun. The Greeks were also the first to describe the electrostatic properties of amber.

Ancient Romans loved amber as well. From the writings of **Thales of Miletus** it appears that Westerners knew as long ago as 600 B.C. that amber becomes charged by rubbing.

There was little real progress until the English scientist **William Gilbert in 1600** described the electrification of many substances and coined the term electricity from the Greek word for amber. As a result, Gilbert is called the "father of modern electricity."

One of nature's most spectacular displays of electricity is the lightning observed during a thunder storm. **Benjamin Franklin (1706-1790)** determined that electricity originates from charges, positive or negative.

We know now that all material bodies possess electric charges. Electrons carry negative charges while protons carry positive charges. The electric force that stationary objects exert on each other is called the **electrostatic force**. This force depends upon the distance between the two point charges and the amount of charge on each.

Experiments have demonstrated that the greater the charge and the closer they are to each other, the greater the force. If charges have unlike signs, each charge is attracted to one other, whereas like charges repel each other.

These attractive forces and repulsive forces act along the line between the charges, and are equal in magnitude but opposite in direction (in accordance with Newton's 3rd law).

The French physicist **Charles-Augustin Coulomb (1736-1806)** experimented with electric force between two point charges (the unit of charge is the Coulomb, C). His work resulted in a law. **Coulomb's Law** states:

"The magnitude of the electrostatic force (F), exerted by one point charge on another point charge is directly proportional to the magnitudes of the two point charges, and inversely proportional to the square of the distance (r) between the charges."

For a pair of charges q_1 and q_2, separated by a distance r, Coulomb's Law may be stated as follows:

$\mathbf{F = k(q_1q_2/r^2)}$ ....**Inverse-Square Law**

The constant of proportionality, k, in Coulomb's Law is, $\mathbf{k = 8.99 \times 10^9\ Nm^2/C^2}$.

Such a force is transmitted by the presence of an electric field. We can **define an electric field** as a region around a charge where a force is exerted on another charge. Since like charges repel, and unlike charges attract, the convention adopted is that electric field vectors always point **away from a positive and toward a negative** charge.

The **electric field strength, E,** can be determined by probing a region of space with a test charge, q, and noting the force per unit charge. This would then give the equation, $\mathbf{E = F/q}$, where F is the force given by Coulomb's Law.

Since the electric field was "set-up" by some charge Q, we can state Coulomb's Law as $\mathbf{F = k(Qq/r^2)}$, where r is the distance from Q to q.

Substituting for F in the equation E = F/q, we get

$\mathbf{E = k(Q/r^2)}$ ....**Inverse-Square Law**

Electric force and electric field are vectors. Hence, they have magnitudes and directions.

The **Principle of Superposition** also applies to the electric fields produced by multiple charges. That is, the net electric field at a point due to several charges is the vector sum of the electric fields due to the individual charges.

For example, when **more than two charges** are present, the net force on any one charge is equal to the **vector sum** of each of the forces produced by other charges.

In other words, the force on charge q_1 due to the presence of charges q_2 and q_3, is the superposition of the forces exerted by q_2 and q_3. That is, the net force F on charge q_1 is,

$\mathbf{F_{net} = F_{12} + F_{13}}$ ....**Vector Equation**

where, F_{12} is the force on q_1 due to the presence of charge q_2, and F_{13} is the force on q_1 due to q_3.

While solving a problem, it is useful to rewrite the vector equation in its **component form** as follows:

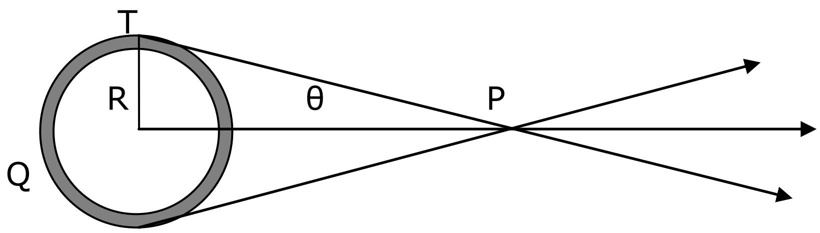
$\mathbf{F_x = (F_{12})_x + (F_{13})_x}$ $\qquad$ $\mathbf{F_y = (F_{12})_y + (F_{13})_y}$

$\mathbf{F_{net} = [(F_x)^2 + (F_y)^2]^{1/2}}$

$\mathbf{\theta = tan^{-1} [F_y/F_x]}$

We will also use **Integral Calculus** to determine the electric field, E, due to various charge distributions.

For example, consider the E-field at point P on the axis of a ring of radius R, carrying total charge Q spread uniformly around the ring.



The charge density, **λ = Q/length,** on the ring is just the total charge Q on the ring divided by 2πR, or, **λ = Q/2πR**. Let **PT = r**, and **PR = x**.

Now, consider a **differential charge, dq,** on a ring element, ds. Therefore **λ = dq/ds**.

Treating the differential charge dq as a point charge, the **differential electric field** at a distance x from the center of the ring is:

$dE = k\, dq/r^2$ with r = distance from P to T

At point P, a distance x from the center of the ring, the y components of the E-field cancel.

The **net E-field, E_x**, is along the axis, away from the ring since the charge Q is positive.

$E_x = \int k\, dq/r^2 \cdot \cos(\theta)$, where $\cos(\theta) = x/r$

$E_x = \int k\, \lambda ds\, dq/r^2 \cdot x/r$, integrate from 0 to 2πR

$E_x = k\, \lambda x/r^3 \cdot 2\pi R$, but $\lambda = Q/2\pi R$

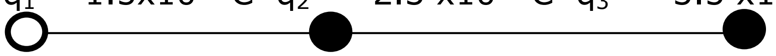
$E_x = k\, Qx/r^3 = kQx/\sqrt{(x^2+R^2)}^3$

At the **center of the ring, x = 0**, and hence **E = 0**. Very far from the ring, **x ›› R**, and E reduces to that of a point charge, or, **$E = k\, Q/x^2$** .

Here is a solved problem on Coulomb's Law:

Three charges are arranged as shown below. Determine the net force on charge q^3.

$q_1 = 1.5x10^{-7}$ C $q_2 = -2.3 \, x10^{-7}$ C $q_3 = -3.5 \, x10^{-4}$ C



$r_1 = 1.4$ m $r_2 = 1.7$ m

Solution: Find the force, F_{31}, on charge q_3 due to q_1, then find the force, F_{32}, on charge q_3 due to q_2. After that, use Superposition.

$F_{NET} = F_{31} + F_{32} = kq_3q_1/(r_1+r_2)^2 + kq_3q_2/r_2^2$

$F_{NET} = kq_3[q_1/(r_1+r_2)^2 + q_2/r_2^2]$

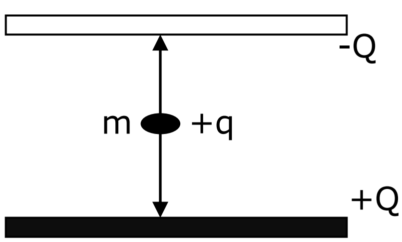
$F_{NET} = 8.99x10^9(-3.5 \, x10^{-4})[(1.5x10^{-7}/3.1^2) +$

$(-2.3 \, x10^{-7}/1.7^2)]$ **= .20 N, to the right**

Here is a solved problem on Electric Fields:

Consider a charge, $q = 3.2x10^{-6}$ C, with a mass, $m = 1.7x10^{-5}$ kg, in equilibrium midway between two oppositely charged, parallel metal plates. Calculate the electric field strength between the plates.

Solution: Since the charged mass is in equilibrium, the forces on it, electrostatic and gravitational, must be equal.



upward force is F_E

downward force is F_G

Since the charged mass is in equilibrium, $F_E = F_G$
$F_E = qE$ and $F_G = mg$ therefore, $qE = mg$

$E = mg/q = (1.7x10^{-5}kg \cdot 9.8m/s^2)/3.2x10^{-6}$ C

= **52N/C**

Chapter 2 - Gauss' Law.

This chapter is devoted to the work of one of the greatest mathematicians of all time. **Carl Friedrich Gauss (1777-1855)** influenced the work in many areas of mathematics from number theory and geometry through differential equations. He also made equally significant contributions to theoretical physics.

When you study this chapter, you should be able to:

- define and compute electric flux through a surface
- use Gauss' Law and the concept of Gaussian surfaces to find the flux, electric field, or total charge enclosed by the surface
- describe and explain the properties of a conductor in electrostatic equilibrium.

Electric flux, Φ (Phi), is proportional to the number of electric field lines through an area and calculated by the Dot Product, $\mathbf{\Phi = E \cdot A = EA\cos(\theta)}$. Realize that flux is a scalar.

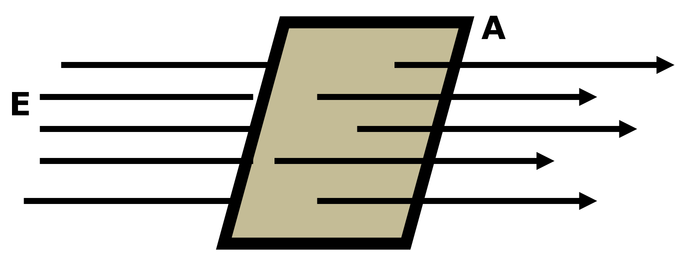
Each line represents the force per unit charge on a test charge placed at a point in the field.

Electric flux through a surface is defined as the dot product of the electric field and the "vector area" of the surface, where the "vector area" points in the direction of the **normal to the surface**.

Units of electric flux are $N \cdot m^2/C$. Flux going INTO a closed surface is negative; flux coming OUT OF a closed surface is positive.

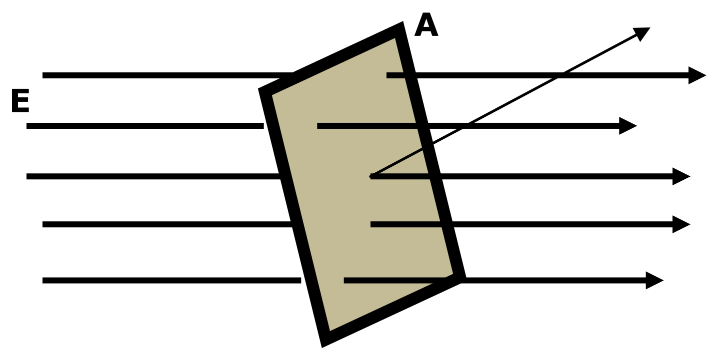
We can visualize electric flux in this way:

(a) If the field lines are perpendicular ($\theta = 0$) to the area that they intersect, and the field is constant,



then the **flux is E times A, or EA**,

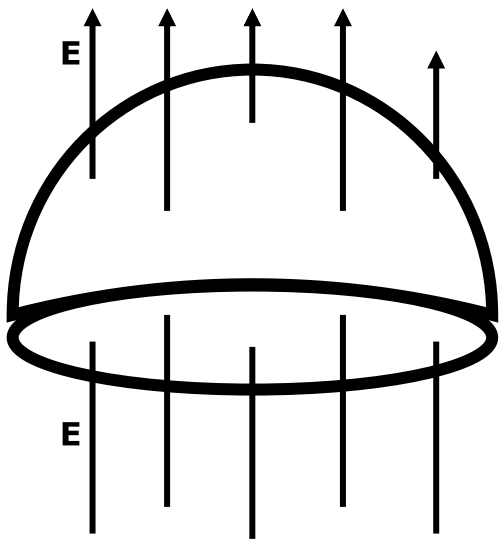
(b) If the field lines are not perpendicular to the area, but the field is constant,



then the **flux is given by EAcos(θ)**, where θ is the angle between the perpendicular to the area, A, and the field vector.

If the field is non-uniform, we'll need an integral given by, **$\Phi = \int E \cdot dA$**. This is a Surface Integral where the differential, dA, represents a tiny patch of the closed surface. Taking an infinite sum, we get the total flux.

Consider an example of trying to find the **flux through a hemisphere**. This would be very difficult if we tried to sum the dot product contributions over the entire curved surface.



To find the flux through the hemisphere, consider the circular bottom of radius r. This circle "captures" all the field lines that also pass through the hemisphere.

Therefore, **$\Phi = \int E \cdot dA = E\int dA$**, with the surface integral **$\int dA$** = **πr^2** . So, for the flux through the hemisphere, we have **$\Phi = \pi r^2 E$**.

We will also need the following charge density equations for lambda, sigma, and rho:

- **Linear charge density is given by $\lambda = q/L$**
- **Area charge density is $\sigma = Q/A$**
- **Volume charge density is $\rho = Q/V$.**

Gauss' Law

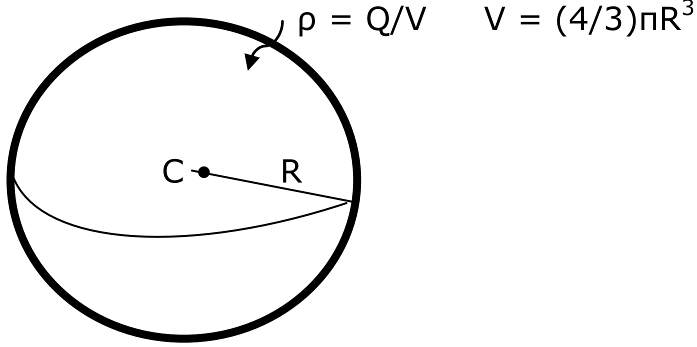
The net electric flux through a closed surface (called a Gaussian surface) is given by the enclosed charge q divided by the permittivity constant epsilon nought, ε_o . $\boldsymbol{\int E\cdot dA = Q_{enc}/\varepsilon_o}$

$\boldsymbol{\varepsilon_o = 1/4\pi k = 8.85\times10^{-12} C^2/Nm^2}$

According to Gauss, we can use either side of the law to calculate the electric flux.

Also, Gauss' Law makes it easier to calculate the electric field at any distance from a charge if we enclose the charge with a Gaussian Surface.

For example: Suppose there's a solid insulating sphere of radius R containing a uniform volume charge density, $\rho = Q/V$, and you need to find the electric field inside, at $r_1<R$, and outside, at $r_2>R$.



For $\boldsymbol{r_1<R}$, consider a spherical Gaussian surface centered at C, of radius r_1.

$\int E\cdot dA = Q_{enc}/\varepsilon_o$ $\rho = Q/V$ with $Q_{enc} = \rho V$

$E\cdot 4\pi r_1^2 = \rho(4/3)\pi r_1^3/\varepsilon_o$ $\boldsymbol{E = \rho r_1/3\varepsilon_o}$

For $\boldsymbol{r_2>R}$, consider a spherical Gaussian surface centered at C, of radius r_2.

$E\cdot 4\pi r_2^2 = \rho(4/3)\pi R^3/\varepsilon_o$ $\boldsymbol{E = \rho R^3/3\varepsilon_o r_2^2}$

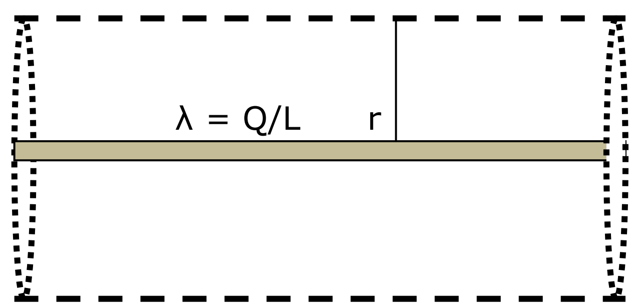
At the surface, $\boldsymbol{r_1 = r_2 = R}$, $\boldsymbol{E = \rho R/3\varepsilon_o}$

Properties of a Conductor:

- The electric field is zero everywhere inside the conductor
- Any charge placed inside the conductor will move immediately to its surface
- The electric field just outside the conductor is perpendicular to the surface at that point
- On an irregularly shaped conductor, charge tends to accumulate where the radius of curvature is smallest (at sharp points, like a lightning rod).

Here is a sample problem using Gauss' Law:

Find an expression for the electric field at a distance r from a line of charge with linear charge density λ.



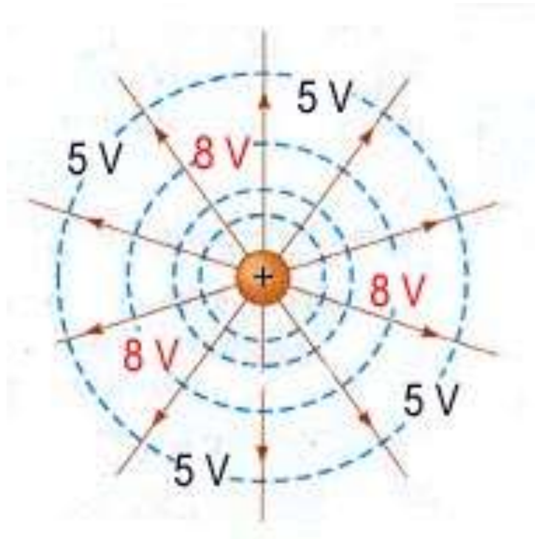
Enclosing the line of charge with a cylinder of radius r and length L, we can use Gauss' Law.

$\int E \cdot dA = Q_{enc}/\varepsilon_o$ $\lambda = Q/L$ with $Q_{enc} = \lambda L$

$E \cdot 2\pi r \cdot L = \lambda L/\varepsilon_o$

$\mathbf{E = \lambda / 2\pi\varepsilon_o r}$

Chapter 3 - Electric Potential.



We know that an electric field is a region in space where an electric force is exerted on a charge. Electric lines of force represent the direction that a positive test charge would move in an electric field. By convention, they originate at positively charged objects and terminate at negatively charged objects.

Therefore, when a positive test charge, q_o, is moved between points A and B in an electric field, **E**, work is done resulting in a change in the potential energy of the charge-field system. $\mathbf{W_{AB} = \Delta U = U_A - U_B}$.

This change is also given by Integral Calculus with, $\mathbf{\Delta U = -q_o \int E \cdot ds}$, evaluated from A to B.

We then say that these two points differ in their electric potential, V, with $\mathbf{V = U/q_o}$. The electric potential difference (V) is the work done per unit charge as a charge is moved between two points in an electric field. Simply stated, $\mathbf{W = U = q \cdot V}$.

We can easily calculate the electric potential difference between two points, A and B, in an electric field as $\mathbf{V_B - V_A = U_A/q_o - U_B/q_o = -W_{AB}/q_o}$.

Now, the Law of Conservation of Energy can be stated as $\mathbf{K_A + qV_A = K_B + qV_B}$.

The Volt (V) is the unit used to measure electric potential difference. It is named after **Alessandro Volta (1745–1827)** an Italian physicist known for inventing the battery, "Voltaic Pile."

Since a Volt measures work done per unit charge, a **Volt = Joule/Coulomb**. Other equivalent expressions for a Volt exist.

"Potential" is shorthand for change in electric potential, or potential difference.

An electric potential difference must exist for current to flow in an electric circuit. Current always flows from high to low potential.

The potential difference ΔV between two points A and B in an electric field E is defined as

$\Delta \mathbf{V} = \Delta \mathbf{U}/\mathbf{q_o} = -\int \mathbf{E}\cdot d\mathbf{s}$, evaluated from A to B.

The potential of the earth is arbitrarily said to be zero. An object connected directly to the ground can be described as being "grounded". (The original expression was "earthed".)

A ground may also be a common plane of zero voltage compared with the rest of the circuit.

The potential at any point in an electric field can be either positive or negative with respect to the earth, depending on the nature of the charge.

The potential difference between two points A and B in a uniform electric field **E** is **V = -Ed**. Here d = |**s**| where **s** is a vector that points from A to B and is parallel to **E**.

Consider a charge placed in an electric field. Let us select some arbitrary reference point in the field, at which the electric potential energy of the charge is defined to be zero.

This uniquely defines the electric potential energy of the charge at every other point in the field.

We know that the electric potential energy is simply the work done in moving the charge from one point to another along any path, and can be calculated using **$V = E \cdot d$** or **$V = kq/r$** .

It should be clear that potential depends on both the particular charge which we place in the field and the magnitude and direction of the electric field along some arbitrary route between points A and B.

However, it is also clear that it is directly proportional to the magnitude of the charge.

An **equipotential surface** is one on which all points are at the same electric potential. Equipotential surfaces are **parallel to** electric field lines.

If we define V = 0 at $r_A = \infty$, the electric potential due to a point charge at any distance r from the charges is **$V = kq/r$** .

We can obtain the electric potential associated with a **group of point charges** by summing the potentials due to the individual charges, **$V = k\Sigma q_i/r_i$** .

The potential energy associated with a pair of point charges separated by distance r_{12} is **$U = kq_1q_2/r_{12}$** . This represents the work done by an external agent when the charges are brought from an infinite separation to the separation r_{12} .

We obtain the potential energy of a distribution of point charges by summing terms of the above equation over all pairs of particles.

If we know the electric potential as a function of **coordinates x, y, and z**, the components of the electric field are found by taking the negative derivative of the electric potential with respect to the coordinates. For example **$E_x = -dV/dx$** .

The electric potential due to a **continuous charge distribution** is $\mathbf{V = k\int dq/r}$.

Every point on the surface of a charged conductor in electrostatic equilibrium is at the same electric potential. The potential is constant everywhere inside the conductor and equal to its value at the surface.

Here are some solved problems on potential:

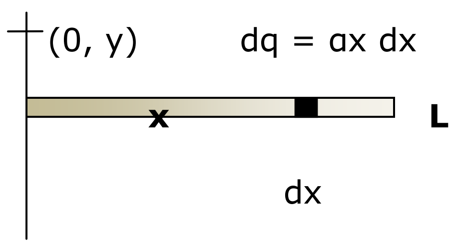
1. A charged particle (q = −8.0 mC) moves in a region where the only force acting on the particle is an electric force. It is released from rest at point A, and at point B the kinetic energy of the particle is equal to 4.8 J. Find the electric potential difference $V_B - V_A$.

Solution: $q = -8.0$ mC $K_A = 0$ $K_B = 4.8$ J

$K_A + qV_A = K_B + qV_B$ $V_B - V_A = -K_B/q$

$V_B - V_A = -4.8\text{ J}/-8.0\times10^{-3}\text{C} =$ **600 V**

2. A charge having a non-uniform linear charge density, $\lambda(x) = \alpha x$, where α (alpha) is a positive constant, is distributed along the x-axis from x = 0 to x = L. Determine the total distributed charge Q. Also find the electric potential at the point (0, y) on the positive y axis.



$dq = \alpha x\, dx$ $Q=\int dq=\int \alpha x\, dx$

integrate from 0 to L

$\mathbf{Q = \alpha L^2/2}$

$V = \int dV$, where $dV = k\,dq/r = k\alpha x\, dx/(x^2+y^2)^{1/2}$

$V = \int k\alpha x\, dx/(x^2+y^2)^{1/2}$, from 0 to L

$\mathbf{V = k\alpha[\sqrt{(L^2+y^2)}-y]}$ since $\mathbf{\alpha = 2Q/L^2}$

$\mathbf{V = (2kQ/L^2)\,[\sqrt{(L^2+y^2)}-y]}$

Chapter 4 - Capacitance.

A capacitor is a device that stores charge. Capacitors are formed by a pair of conductors separated by an insulator. They are found in computer keyboards, automobile ignition systems, and flash cameras.

The type of capacitor we are most interested in will have a charge Q and -Q on each conductor. There will also be a resultant voltage, V, between the two conductors.

This voltage is linearly dependent on the charge. If we triple the charge, we triple the voltage. Because of this relationship, the **ratio of Q/V is a constant** for a capacitor.

The value of Q/V for a given capacitor is known as its **capacitance**. This gives the equation, **C = Q/V** .

The unit of capacitance is the Farad, after **Michael Faraday (1791-1867)**. It is equivalent to one coulomb per volt.

One Farad is an extremely large capacitance; most capacitors come in units of micro (μ), nano (n), or pico (p) farads.

The capacitance of a capacitor is determined by two factors:

- geometry of the capacitor
- material between the conductors.

This material is known as a dielectric.

In a parallel plate capacitor, capacitance can be calculated by using the equation,

$C = \varepsilon_o A/d$, $\varepsilon_o = 8.85 \times 10^{-12} C^2/Nm^2$,

ε_o is the permittivity of free space, A is the **area** of a plate, and d is the **distance** between the plates.

When one has several capacitors in a circuit, they can be combined in many ways. There are equations which show how to calculate the **equivalent capacitance, C_{eq}** for any type of combination.

For example, to find the equivalent capacitance of **two or more capacitors in series**, you would add the inverse of their values and then take the inverse of their sum.

The equation for **n capacitors series (same Q, different V)** is given by:

$1/C_{eq} = 1/C_1 + 1/C_2 + \ldots + 1/C_n$

For the equivalent capacitance of **n capacitors in parallel (same V, different Q)** you would add their capacitances.

$C_{eq} = C_1 + C_2 + \ldots + C_n$.

In circuit applications, the capacitor can be subjected to numerous electrical, mechanical, and environmental stresses.

One of the most noticeable effects of these stresses is the phenomena of capacitance variation. The insertion of a non-conducting material (dielectric) between the plates is an easy way to increase capacitance.

This increase is based on the value of **κ, "kappa" , the dielectric constant**.

"C" varies directly with "A" and "κ" and inversely with "d" or $\mathbf{C = \kappa\varepsilon_o A/d}$.

Any change in "C" must come as a result of some change or combination of changes in "A", "κ", or "d".

The value of "A" is set by design and, once a capacitor is made, it is almost impossible for "C" to change due to a change in "A". This is not a normal factor in capacitance variation.

The value of "d" is also set by design. Some small changes in "d" can occur due to external or internal pressure changes resulting in mechanical movement of the electrodes. This is not usually critical nor does it result in any large variations.

The value "κ" is also initially set by design in the choice of dielectric material used to make the capacitor.

For example:

Material	**Constant(κ)**	**Strength(V/m)**
Vacuum	1.00000	-
Air	1.00059	$3x10^6$
Paper	3.7	$16x10^6$
Teflon	2.1	$60x10^6$

Many factors will cause the "κ" to change, and this change in "κ" will vary for different materials.

The "κ" in the basic formula is the effective dielectric constant of the total "space" between the electrodes.

This "space" will consist of the dielectric material (or materials if a multiple dielectric design), air, impregnant (if an impregnated unit), and even moisture (if present).

The phenomenon of capacitance is a type of electrical charge and energy storage in the form of a field in an enclosed space.

As charge is placed on the plates, the potential, V, increases, or $dV = dq/C$).

To derive an expression for the potential energy stored in a charged capacitor, we must use Integral Calculus to account for all the little charges "dq" being added to a plate.

We know that $dU = q \cdot dV = q\, dq/C$.

Therefore, $U = \int dU = \int q \cdot dV = \int q\, dq/C = \int q/C\, dq$

$U = 1/C \int q\, dq$ or, $\mathbf{U = Q^2/2C}$

Since $Q = CV$, we get $\mathbf{U = CV^2/2}$ and $\mathbf{U = \frac{1}{2}QV}$.

So the energy stored in a capacitor can be thought of as the potential energy stored in the system of positive charges that are separated from the negative charges, much like a stretched spring has potential energy associated with it.

Here's **another way** to think of the energy stored in a charged capacitor:

If we consider the space between the plates to contain the energy (equal to $\mathbf{\frac{1}{2}CV^2}$) we can calculate an **energy density, u,** (Joules per unit volume).

The volume between the plates is the area times the plate separation, or $A \cdot d$.

The energy density is $\mathbf{u = (\frac{1}{2}CV^2)/Ad = \frac{1}{2}e_o\, E^2}$.
(Recall that $C = e_o\, A/d$ and $V = Ed$.)

Here are some solved problems on capacitors:

1. A 15 μF capacitor is connected in series to a parallel arrangement of a 10 μF and a 20 μF. The entire circuit is placed across a potential difference of 18 V. Determine the energy stored in the 10 μF.

Solution: Let C_1 = 15 μF C_2 = 10 μF C_3 = 20 μF
Equivalent capacitance for the parallel combination of C_2 and C_3 is $C_{eq} = C_{23} = C_2+C_3$ = 30 μF, in series with C_1. Now, C_{23} and C_1 have same charge, Q, but different voltage. C = Q/V   or V = Q/C

$18V = Q/C_1 + Q/C_{23} = Q/15\mu F + Q/30\mu F$

$18V = 3Q/30\mu F$ Q = .18 mC

Voltage across C_1, is V_1 = .18 mC/15 μF = 12 V
This means that the voltage across C_{23} = 6 V ,
Energy in C_2, $U = \frac{1}{2}CV^2 = \frac{1}{2}\cdot 10\mu F\cdot(6V)^2$ = **.18 mJ**

2. Consider the Earth and a cloud layer 800 m above the Earth as the "plates" of a capacitor. Calculate the capacitance if the cloud layer has an area of 1.0 km^2. If an electric field of 2.0×10^6 N/C makes the air break down and conduct electricity (lightning), what is the maximum charge the cloud can hold?

Solution: d = 800 m   A = 1.0 km^2 = 1.0×10^6 m^2
E = 2.0×10^6 N/C   C = _____   Q = _____

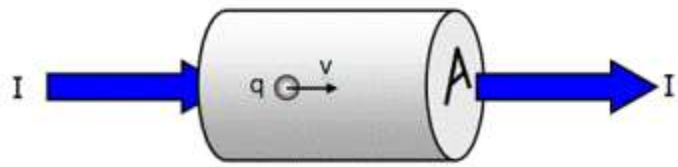
$C = \varepsilon_o A/d = 8.85\times10^{-12} C^2/Nm^2\cdot(1.0\times10^6\ m^2)/(800\ m)$

C = 11.1 nF

C = Q/V and V = Ed, therefore C = Q/(Ed)

$Q = EdC = 2.0 \times 10^6\ N/C\cdot(800\ m)(11.1\ nF)$ = **17.8 C**

Chapter 5 - Current and Resistance.



In electricity two fundamental concepts are current and voltage. For any electrical element the **voltage** (V) across the element is the potential difference between its two ends, while the **current**, I, (defined as **$I=dQ/dt$**) through the element is the rate at which electrical charges are flowing.

For many devices (but not all) the voltage and the current are proportional to each other, and we can write **$I = (1/R)\cdot V$** in which R is a constant of proportionality known as the **resistance**.

The current in a conductor is related to the motion of charge carriers through the equation **$I=nqv_dA$**, where n is the density of charge carriers, and v_d, the **drift velocity**.

Current density, J, can be calculated three ways with **$J=I/A$**, **$J=nqv_d$**, and **$J=\sigma E$**. The quantity, σ, is referred to as **conductivity**.

We can also calculate resistance, R, with **$R = \rho L/A$** with ρ, rho, being the **resistivity**. (**$\rho = 1/\sigma$**)

Resistivity also has a **temperature dependence** which is given by **$\rho=\rho_o[1 + \alpha\cdot(T - T_o)]$**, with α being a **temperature coefficient**.

The equation, **$V = I\cdot R$** is known as **Ohm's Law**, and devices which obey Ohm's Law are known as **linear or ohmic devices**.

Familiar examples are resistors which are found in radios, TV sets, computers, and other electronic systems; the filaments of light bulbs; and the heating elements of electrical ovens.

There are other devices which do not obey Ohm's Law, semiconductor devices such as transistors and diodes, and fluorescent light bulbs. These are known as **nonlinear devices**.

Ohm's Law can be used to solve **simple circuits**. A complete circuit is one which is a closed loop. It contains at least one source of voltage (thus providing an increase in potential energy) and at least one potential drop i.e., a place where potential energy decreases.

If a potential difference (voltage) is maintained across a resistor, the **power**, can be calculated with $\mathbf{P = V \cdot I = I^2 \cdot R = V^2/R}$.

An increase of potential energy in a circuit causes a charge to move from a lower to a higher potential (ie. voltage). Note the difference between potential energy and potential.

Because of the electrostatic force, which tries to move a positive charge from a higher to a lower potential, there must be another "force" to move charge from a lower potential to a higher inside the battery.

This so-called force is called the **electromotive force**, or **emf**. The SI unit for the emf is the volt (and thus this is not really a force, despite its name).

We will use the symbol, $\mathcal{E}$, to represent the emf and Ohm's Law can be expressed as $\mathcal{E} = IR$.

A decrease of potential energy can occur by various means. For example, heat lost in a circuit due to some electrical resistance could be one source of energy drop.

Here are some solved problems dealing with Current and Resistance.

1. Calculate the drift velocity of the charge carriers in a sample of semi conductor which is 5 mm wide and 2 mm thick if the current is 10 mA and the charge carrier density is $6x10^{23}$ m^{-3}.

Solution: $n = 6x10^{23}\ m^{-3}$ $I = 10\ mA = 1x10^{-2}\ A$

$A = 5\ mm \times 2\ mm = 10\ mm^2 = 1x10^{-5}\ m^2$

$q = 1.6x10^{-19}\ C$ $J = I/A = nqv_d$ $v_d = I/nqA$

$v_d = 1x10^{-2}\ A/(6x10^{23}\ m^{-3} \cdot 1.6x10^{-19}\ C \cdot 1x10^{-5}\ m^2)$

v_d = **0.01 m/s**

2. Find the resistance of a 2.5 m length of copper wire with a cross-sectional area of $1.3x10^{-4}\ cm^2$. The resistivity of copper is $1.7x10^{-8}\ \Omega m$.

Solution: $L = 2.5\ m$ $\rho_{cu} = 1.7x10^{-8}\ \Omega m$

$A = 1.3x10^{-4}\ cm^2 = 1.3x10^{-8}\ m^2$ $R = \rho L/A$

$R = (1.7x10^{-8}\ \Omega m \cdot 2.5\ m)/\ 1.3x10^{-8}\ m^2$ = **3.3 Ω**

3. A high-voltage transmission line carries 1000 A at 700 kV for a distance of 100 miles. If the resistance per length in the wire is 0.5 Ω/mile, what is the power loss due to resistive losses

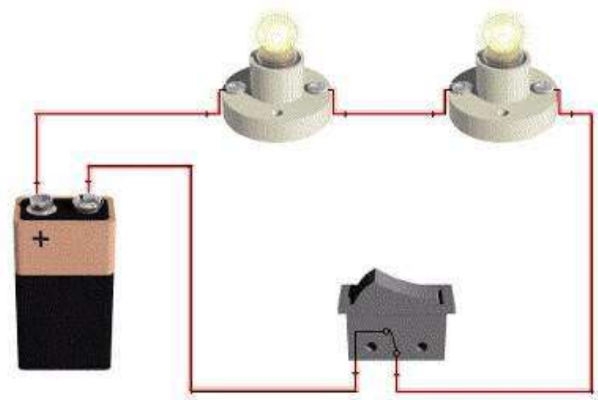
Solution: $R/L = 0.5\ \Omega/mile$ $I = 1000\ A$

$V = 700\ kV = 7x10^5\ V$ $L = 100\ miles$ $P = I^2R$

$P = I^2(R/L) \cdot L = (1000\ A)^2 \cdot (0.5\ \Omega/mile \cdot 100\ miles)$

P = **50 MW**

Chapter 6 - DC Circuits.



The source of electric energy that causes charges to move in electric circuits is the **emf, Ɛ**, with **Ɛ** = IR . Historically such energy sources were called electromotive force, however, it is **not** a "force" but a potential energy per unit charge, or a voltage.

A good example of such a source of electric energy is a **battery**. When a battery is placed in a circuit loop with other circuit elements, such as capacitors or resistors, a DC current flows.

The internal chemistry of the battery provides some internal voltage, or **emf**, for the battery. The actual **terminal voltage** of the battery will be somewhat less due to the voltage drop over the **"internal resistance."**

The expression describing terminal voltage is given by: **V = Ɛ - Ir** . Here, V is the terminal voltage (measured between the two terminals), **Ɛ** is the true emf of the battery, I is the current being drawn from the battery, and r is the battery's internal resistance.

We find that: **P = V(q/t) = VI**. For a resistor, this expression can be rewritten as: **P = (IR)I =** $\mathbf{I^2R}$.

Consider a **single-loop circuit**, one with a single path for current. If we keep track of the voltage around the loop, and remember that voltage (or potential, or electric potential) is potential energy per unit charge, we note that the sum of the potential changes (or potential "drops") around the loop must be zero.

We can summarize **Kirchhoff's Loop Rule** as:

"The sum of the potential changes around a closed path is zero. $\mathbf{\Sigma \mathcal{E} - \Sigma Ir = 0}$."

This rule can be for a simple loop, or for any closed loop in a more complex circuit, say one that has two or three loops.

We can summarize **Kirchhoff's Point Rule** as:

"The algebraic sum of the currents that enter a junction is zero. $\mathbf{\Sigma I = 0}$."

This rule along with the Loop Rule will enable us to analyze various electric circuits.

For **resistors in series**, we use **simple addition**, $\mathbf{R_{EQ} = R_1 + R_2 + \ldots + R_n}$. **(Same I, diff. V.)**

For **resistors in parallel**, we use **reciprocals**, and now have : $\mathbf{1/R_{EQ} = 1/R_1 + 1/R_2 + \ldots + 1/R_n}$. **(Same V, diff. I.)**

The rules for assigning signs to the **voltage changes across capacitors** in a closed loop for Kirchoff's loop rule are:

- $V_c = - Q/C$ if the direction of the loop crosses the capacitor from its positive to its negative plate (high to low)
- $V_c = + Q/C$ if the direction of the loop crosses the capacitor from its negative to its positive plate (low to high)

RC circuits are circuits that contain both **resistors and capacitors**. In a DC circuit (or steady-state circuit, or constant current circuit), a capacitor acts like an open switch.

Its steady-state voltage will be **V = Q/C** and will be equal to the voltage of the battery. However, when the switch is first closed for the circuit, charge must flow until the capacitor is charged. This is called a **transient current**.

Consider a simple loop circuit with a battery with terminal voltage, Ɛ = V_{bat}, a resistor R, a capacitor C, and a switch.

Just before the switch is closed the current is zero. Just after it is closed a current flows and the capacitor starts to charge.

Its voltage will be given by **V = q/C**. In the time q grows to Q until **Q/C = Ɛ**, the current will be zero.

Let us write Kirchhoff's Loop Rule for the circuit as: **Ɛ - IR - q/C = 0** . Note that the current I can be written as **I = dq/dt**.

The expression becomes: **Ɛ - (dq/dt)R - q/C = 0** .

The solution found by separation of variables and integral calculus is: $q(t) = (C\mathcal{E})(1 - e^{(-t/RC)})$.

Note that after a long time $e^{(-t/RC)}$ becomes equal to 0, and the charge is constant.

If we take the derivative of q(t) with respect to t, we get the expression: $I = dq/dt = (\mathcal{E}/R)e^{(-t/RC)}$.

We see that after a long time, the current is zero. The quantity RC is called the **time constant** and is given by the Greek letter tau, **τ**. Here, **τ = RC** and has units of time. It determines how fast a capacitor charges and discharges. After a time interval equal to one time constant **τ** has passed, the **charge is 63.2%** of the maximum value of CƐ.

If we start with a charged capacitor, and have only a resistor and an open switch in the circuit, we can then discharge the capacitor by closing the switch.

Kirchhoff's Loop Rule equation is: $\mathbf{\mathcal{E} - IR - q/C = 0}$. Again, using $\mathbf{I = dq/dt}$, and substituting we now have: $\mathbf{\mathcal{E} - (dq/dt)R - q/C = 0}$.

Deriving, as before, we get: $\mathbf{q(t) = Q_o e^{(-t/RC)}}$. Here $\mathbf{Q_o = C\mathcal{E}}$. Again, using $\mathbf{I = dq/dt}$, we can show that $\mathbf{I(t) = (Q_o/RC)e^{(-t/RC)} = (\mathcal{E}/R)e^{(-t/RC)}}$.

The charge decreases to zero as the capacitor is completely discharged. But, when $\mathbf{t = \tau}$, we get the value that $\mathbf{e^{(-t/RC)} = e^{-1} = 0.37}$, or the charge on the capacitor is down to just over one-third of the original value.

Here are some solved problems on DC circuits:

1. A 5000 Ω resistor, a 50 μF capacitor, and a switch are connected in series with a 6 V battery. The capacitor is initially uncharged. What is the current in the circuit at the instant the switch is closed, t = 0? At t = 0.5 s? What is the maximum charge stored on the capacitor?

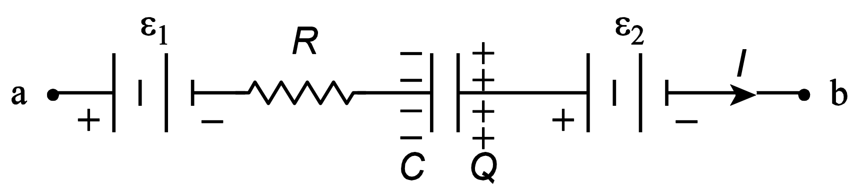
Solution: $R = 5000\ \Omega$ $\quad C = 50\ \mu F = 5\text{x}10^{-5}\ F$
$V = 6\ V$ $\quad \tau = RC = 5000\ \Omega \cdot 5\text{x}10^{-5}\ F = .25\ s$

At t = 0, $I = V/R = 6\ V/5000\ \Omega$ = **1.2 mA.**

At t = 0.5 s, $I = (\mathcal{E}/R)e^{(-t/RC)} = (1.2\text{x}10^{-3}\ A)\cdot e^{(-.5/.25)}$
I = **.162 mA**

For Q_{max}, $Q = C\mathcal{E} = 5\text{x}10^{-5}\ F \cdot 6\ V$ = **300 μC**

2. In the figure, R = 3.0 kΩ, C = 6.0 nF, $\mathcal{E}_1$ = 10 V, Q = 18 nC, $\mathcal{E}_2$ = 6.0 V, and I = 5.0 mA. What is the potential difference $V_b - V_a$?



Solution: For $V_b - V_a$, traverse the circuit from b to a

$V_b - V_a$ = -6 V + 18 nC/6 nF - 5 mA·3 kΩ -10 V

$V_b - V_a$ = **-28 V**

3. In an RC circuit, how many time-constants must elapse if an initially uncharged capacitor is to reach 80% of its final potential difference?

Solution: $q = (C\mathcal{E})(1 - e^{(-t/RC)})$. $v = \mathcal{E}(1 - e^{(-t/RC)})$

For v = .8 $\mathcal{E}$, we have $.8\,\mathcal{E} = \mathcal{E}(1 - e^{(-t/RC)})$

$.8 = 1 - e^{(-t/RC)}$ $\ln(.2) = \ln(e^{(-t/RC)})$ $-t/RC = -1.6$

t = **1.6 RC**

4. Four 1.5-volt AA batteries in series are used to power a transistor radio. If the batteries hold a total charge of 240 C, how long will they last if the radio has a resistance of 200 Ω ?

Solution: V = 4·1.5 V= 6 V Q = 240 C R = 200 Ω
V = IR and I = Q/Δt

Δt = QR/V = 240 C·200 Ω/6 V = 8×10^6 s = **2.2 hr**

Chapter 7: Magnetic Effects.

Magnetic fields exert force on charged particles travelling in them. This effect is seen as the beautiful auroras, for example, in the upper latitudes of the northern hemisphere.

The force, F, produced by a magnetic field on a single charge, q, depends upon the speed, v, of the charge, the strength, B, of the field, and the magnitude of the charge.

The equation is **$F = qvB\sin\theta$** , where θ is the smaller angle between v and B.

In vector form, this equation is given by the cross product **F =** q**vxB** . To find the direction of the force, use the **right hand rule**. Point your fingers in the direction of B, and your thumb in the direction of v. The force comes out of your palm.

The unit of magnetic field strength is the **Newton per Ampere·meter**, N/Am, or Tesla (T), named after Nicola Tesla (1856-1940), who was a physicist/engineer/inventor from Croatia. The non-metric unit is the Gauss (G), with **$1\ T = 10^4\ G$**.

If the charged particle moves parallel to the field lines ($\theta = 0$), then the magnetic force on the particle is zero. If a charged particle is moving perpendicular to a uniform magnetic field, the path of the charged particle is an arc (or circle).

The magnetic force is the source of the centripetal force on the charged particle. This relationship can be used to find the radius of the arc when we set the equations equal to one another. $\mathbf{mv^2/r = qvB}$

Since the magnetic force is perpendicular to the velocity of the charged particle, the force does not cause the speed of the particle to change, only its direction. Thus, **no work is done** by the magnetic force on the charged particle.

Ampere found that a force is also exerted on a **current-carrying wire** in a magnetic field. The equation is, **F = BILsinθ,** where B is the magnetic field in Teslas (T), I is the current, L is the length of wire in meters, and θ is the angle.

Only the perpendicular component of *B* exerts a force on the wire.

If the direction of the current is perpendicular to the field (θ = 90), then the force is given by **F = BIL.**

In vector form, if a long straight conductor of length L carries a current I, the force on that conductor when placed in a uniform magnetic field is calculated using the cross-product, **F =** I(**L x B).**

The net magnetic force on any closed loop carrying a current in a uniform magnetic field is zero.

The **magnetic moment** of a current loop carrying a current I is, **μ = IA,** where A is perpendicular to the plane of the loop and has magnitude equal to the area.

The magnetic moment is a measure of the tendency of the current loop to align itself in a magnetic field. For the direction, curl your fingers around the loop in the direction of the current. Your thumb points in the direction of the magnetic moment.

The torque τ on a current loop when the loop is placed in an external magnetic field is given by the equation, $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$.

When a current-carrying conductor is placed in a magnetic field, a potential difference is generated in a direction perpendicular to both the magnetic field and the current.

This potential difference, known as a **transverse voltage**, was first noticed by Edwin Hall (1855-1938) in 1879 and is known as the **Hall Effect**.

Using the equations for Magnetic Force, Electric Field, and drift velocity derived previously, we can show that the Hall Voltage measured across a conductor of width d and cross-sectional area A is, $\mathbf{\Delta V_H = IBd/nqA}$.

Applications of the Hall Effect are found in sensors or probes used as magnetometers.

Here are solved problems on Magnetic Effects:

1. An electron has a velocity of 6.0×10^6 m/s in the positive x direction at a point where the magnetic field has the components, $B_x = 3.0$ T, $B_y = 1.5$ T and $B_z = 2.0$ T. Find the magnitude of the acceleration of the electron at this point.

Solution: $v = 6.0 \times 10^6$ m/s $B_x = 3.0$ T $B_y = 1.5$ T
$B_z = 2.0$ T $m = 9.11 \times 10^{-31}$ kg $q = 1.6 \times 10^{-19}$ C
$B_{net} = \sqrt{(B_y^2 + B_z^2)} = 2.5$ T

$F = ma = qvB\sin\theta$ $\theta = 90^\circ$ $ma = qvb$ $a = qvB/m$
$a = (1.6 \times 10^{-19}\ \text{C} \cdot 6.0 \times 10^6\ \text{m/s} \cdot 2.5\ \text{T})/9.11 \times 10^{-31}$ kg
$a = \mathbf{2.6 \times 10^{18}\ m/s^2}$

2. An electron moving in the positive x direction experiences a magnetic force in the positive z direction. If $B_x = 0$, what is the direction of the magnetic field?

Solution: -y direction by the right hand rule

3. A charged particle (mass = m, charge = $q > 0$) moves in a region of space where the magnetic field has a constant magnitude of B and a downward direction. What is the magnetic force on the particle at an instant when it is moving horizontally toward the north with a speed v?

Solution: qvB, West, by the right hand rule

4. A circular loop (radius = 0.50 m) carries a current of 3.0 A and has unit normal vector of $(2\mathbf{i} - \mathbf{j} + 2\mathbf{k})/3$. What is the x component of the torque on this loop when it is placed in a uniform magnetic field of $(2\mathbf{i} - 6\mathbf{j})$T?

Solution: r = .50 m I = 3.0 A $\mu = IA = I \cdot \pi r^2 = 3\pi/4$

$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B} = [(3\pi/4)(2\mathbf{i} - \mathbf{j} + 2\mathbf{k})/3] \times (2\mathbf{i} - 6\mathbf{j})$

$\boldsymbol{\tau} = 0 - 3\pi\mathbf{k} + (\pi/2)\mathbf{k} + \pi\mathbf{j} + 3\pi\mathbf{i} = 3\pi\mathbf{i} + \pi\mathbf{j} - (5\pi/2)\mathbf{k}$

$\boldsymbol{\tau}_x = 3\pi\mathbf{i}$ **Nm**

5. What will be the radius of curvature of the path of a 3.0 keV proton in a perpendicular magnetic field of magnitude .80 T?

Solution: $q = 1.6\text{x}10^{-19}$ C $m = 1.67\text{x}10^{-27}$ kg

$K = ½mv^2 = 3.0\text{ keV}\cdot(1.6\text{x}10^{-19}\text{ J/eV}) = 4.8\text{x}10^{-16}\text{ J}$

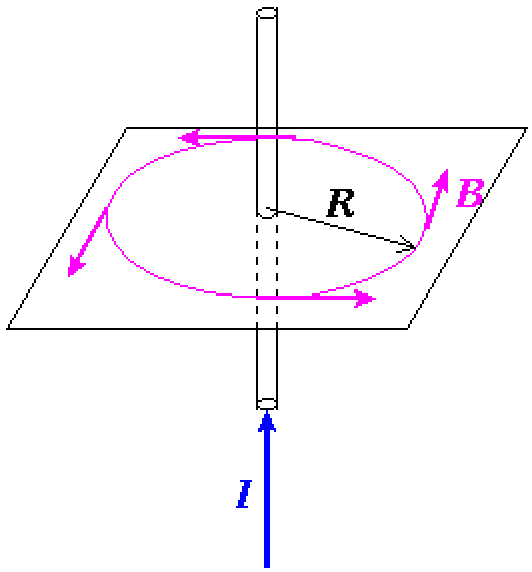
$v = \sqrt{(2K/m)} = \sqrt{(2\cdot 4.8\text{x}10^{-16}\text{ J}/1.67\text{x}10^{-27}\text{ kg})}$

$v = 7.6\text{x}10^5$ m/s $mv^2/r = qvB$ $r = mv/qB$

$r = (1.67\text{x}10^{-27}\text{ kg}\cdot 7.6\text{x}10^5\text{ m/s})/(1.6\text{x}10^{-19}\text{ C}\cdot .80\text{ T})$

r = **9.9 mm**

Chapter 8: Magnetic Field Sources.



Magnetic fields arise from charges, similarly to electric fields, but are different in that the charges must be moving.

The unit of magnetic field strength is the Newton per Ampere·meter, N/Am, or Tesla (T). The non-metric unit is the Gauss (G), with 1 T = 10^4 G. The Tesla is named after **Nicola Tesla (1856-1940)**, physicist/engineer/inventor from Croatia.

A long straight wire carrying a current is the simplest example of a moving charge that generates a magnetic field. We mentioned that the direction of the force a charge felt when moving through a magnetic field depended on the right-hand rule.

The direction of the magnetic field due to moving charges will also depend on the right hand rule.

For the case of a **long straight wire** carrying a current, I, the magnetic field lines wrap around the wire. By pointing one's right thumb along the direction of the current, the direction of the magnetic field can by found by curving one's fingers around the wire.

The strength of the magnetic field depends on the current I in the wire and r, the distance from the wire.

The equation is $\mathbf{B=\mu_o I/(2\pi r)}$, with the constant μ_o, "mu naught", given as $\mathbf{4\pi \times 10^{-7}}$ **Tm/A** .

The constant is the **permeability of free space**. The reason it does not appear as an arbitrary number is that the units of charge and current (coulombs and amps) were chosen to give a simple form for this constant.

If one remembers the case of the electric field of a uniformly charged wire, it also fell off as 1/r.

There is no real analogy to Coulomb's law for magnetism, as the magnetic field of a point charge is complicated since it can't be standing still to generate a magnetic field.

Ampere's Law is the magnetic equivalent of Gauss's Law. It is different in that it refers to a closed loop and the surface enclosed by it (rather than a closed surface and the volume enclosed by it, as is the case with Gauss's Law).

Consider a closed loop, not necessarily a circle, which is broken into small elements of length **dl** , with a magnetic field **dB** at each element.

The sum over elements of the component of the magnetic field along the direction of the element, times the element length, is proportional to the current I that passes through the loop.

This is Ampere's Law. "The line integral $\mathbf{\int B \cdot dl = \mu_o I}$."

For the case of a wire, the loop can be a circle drawn around the wire, and since the field is always tangent to the circle, $\cos(\theta) = 1$.

The circumference of the circle of radius r is $2\pi r$, therefore Ampere's Law becomes: $\mathbf{B \cdot 2\pi r = \mu_o I}$.

Ampere's Law will also allow us to calculate the magnetic field for a solenoid. A solenoid is a coil of wire designed to create a strong magnetic field inside the coil. By wrapping the same wire many times around a cylinder, the magnetic field due to the wires can become quite strong.

Consider a **solenoid of N turns**, or loops. Intuitively we know that more loops will bring about a stronger magnetic field.

Ampere's Law can be applied to find the magnetic field inside a long solenoid as a function of the number of turns per unit length, **n = N/L** , and the current, I. The equation we get is given by,

$\mathbf{B=\mu_o(N/L)I}$.

Only the upper portion of the path contributed to the sum because the magnetic field is zero outside, and because the vertical paths are perpendicular to the magnetic field.

This is the result we have been after. The magnetic field inside a solenoid is proportional to both the applied current and the number of turns per unit length.

There was no dependence on the diameter of the solenoid or even on the fact that the wires were wrapped around a cylinder and not a rectangular shape.

Most importantly, the result did not depend on the precise placement of the path inside the solenoid, indicating that the magnetic field is constant inside the solenoid.

A current-carrying wire acts as a source of magnetic field. A **second wire will feel a force** from the magnetic field of the first one. The force on wire a due to wire b will always be equal and opposite to the force on wire b due to wire a.

Using the right-hand rule one can show the following:

(a) Parallel wires with current flowing in the **same** direction, **attract each other**.

(b) Parallel wires with current flowing in **opposite** directions, **repel each other**.

The equation for this force is: $\mathbf{F/L = \mu_o I_1 I_2 / 2\pi r}$

Biot-Savart Law: The magnetic equivalent of Coulomb's law is the Biot-Savart Law for the magnetic field produced by a short segment of wire, dl, carrying current I: $\mathbf{dB = (\mu_o I/4\pi)\ dl\ sin(\varphi)/r^2}$, where the direction of dl is in the direction of the current and where the vector, r , points from the short segment of current to the observation point where we are to compute the magnetic field.

Since current must flow in a circuit, integration is always required to find the total magnetic field at any point. The constant, μ_o ,is chosen so that when the current is in amps and the distances are in meters, the magnetic field is correctly given in units of Tesla.

A quick comparison of this value with the Biot-Savart Law probably makes you wonder what role 4π is supposed to play here. It plays the same role it did in Coulomb's Law: it was required in Coulomb's law so that Gauss's Law wouldn't have a 4π, and it is required in the Biot-Savart Law so that Ampere's Law won't have one either.

There are two simple cases where the magnetic field integrations are easy to carry out, and fortunately they are in geometries that are of practical use. We use the formula for the magnetic field of an infinitely long wire whenever we want to estimate the field near a segment of wire, and we use the formula for the magnetic field at the center of a circular loop of wire whenever we want to estimate the magnetic field near the center of any loop of wire.

Infinitely Long Wire: The magnetic field at a point a distance r from an infinitely long wire carrying current I has magnitude $\mathbf{B = \mu_o I/(2\pi r)}$, and its direction is given by a right-hand rule: point the thumb of your right hand in the direction of the current, and your fingers indicate the direction of the circular magnetic field lines around the wire.

Circular Loop: The magnetic field at the center of a circular loop of current-carrying wire of radius R has magnitude $\mathbf{B = \mu_o I/(2R)}$, and its direction is given by another right-hand rule: curl the fingers of your right hand in the direction of the current flow, and your thumb points in the direction of the magnetic field inside the loop.

Long Thick Wire: Imagine a very long wire of radius a carrying current I distributed symmetrically so that the current density, J, is only a function of distance r from the center of the wire. Ampere's Law is used to find the magnetic field at any radius r.

Outside the wire, where r >= a, we have the equation that, $\mathbf{B = \mu_o I/(2\pi r)}$, just as if all the current were concentrated at the center of the wire. Now for the region inside the wire, where r < a, we have $\mathbf{B = \mu_o I(r)/(2\pi r)}$, where I(r) is the current flowing through the disk of radius r inside the wire; the current outside this disk contributes nothing to the magnetic field at r.

Long Solenoid: Imagine a long solenoid of length L with N turns of wire wrapped evenly along its length. Ampere's Law can be used to show that the magnetic field inside the solenoid is uniform throughout the volume of the solenoid (except near the ends where the magnetic field becomes weak) and is given by, as before, $\mathbf{B = \mu_o (N/L)I = \mu_o nI}$, where $\mathbf{n = N/L}$.

Toroid: Imagine a toroid consisting of N evenly spaced turns of wire carrying current I. (Imagine winding wire around a donut, with the wire coming up through the hole, out around the outside, then up through the hole again, etc..)

Ampere's Law can be used to show that the magnetic field within the volume enclosed by the toroid is given by $\mathbf{B = \mu_o NI/(2\pi R)}$. In this case, $\mathbf{2\pi R}$ is referred to as the "circumferential length."

Here are problems on Magnetic Field Sources:

1. One long wire carries a current of 30 A along the entire x axis. A second long wire carries a current of 40 A, perpendicular to the xy plane, and passing through the point (0, 4, 0) m. What is the magnitude of the resulting magnetic field at the point y = 2.0 m on the y axis?

Solution: B_1 is the magnetic field of the 30 A wire, and B_2 is the magnetic field of the 40 A wire. Find each with $\mathbf{B = \mu_o I/(2\pi r)}$ and use superposition. So, $B_1 = (4\pi x10^{-7}\ Tm/A\cdot 30\ A)/(2\pi\cdot 2m) = 3.0\ \mu T$. Also, $B_2 = (4\pi x10^{-7}\ Tm/A\cdot 40\ A)/(2\pi\cdot 2m) = 4.0\ \mu T$. Since $B_1 \perp B_2$, B_{net} = **5.0 μT**

2. A segment of wire of total length 2.0 m is formed into a circular loop having 5.0 turns. If the wire carries a 1.2 A current, determine the magnitude of the magnetic field at the center of the loop.

Solution: For each circular loop, $C = 2\pi r = 2.0\ m/5$, or .4 m. Now $r = .2/\pi$ m, and $B = \mu_o I/(2r)$. Therefore, $B = (4\pi x10^{-7}\ Tm/A\cdot 1.2\ A)/(2\cdot .2/\pi\ m)$ Resulting in, B = **59 μT**

3. A long straight wire (diameter = 2.0 mm) carries a current of 25 A. What is the magnitude of the magnetic field 0.50 mm from axis of the wire?

Solution: $B = \mu_o I/(2\pi r)$, R = 1.0 mm, r = .50 mm $I = 25\ A\ (r^2/R^2) = 6.25\ A$.

$B = (4\pi x10^{-7}\ Tm/A\cdot 6.25\ A)/(2\pi\cdot .5\ mm)$ = **2.5 mT**

Chapter 9: Faraday's Law.

Michael Faraday (1791-1867)

Background: Our studies so far have been concerned with electric fields due to stationary charges and magnetic fields produced by moving charges. This chapter deals with electric fields that originate from changing magnetic fields.

Experiments conducted by **Michael Faraday** in England in 1831 and independently by **Joseph Henry** in the United States that same year showed that an electric current could be induced in a circuit by a changing magnetic field.

Faraday discovered that when the magnetic flux, given by the Greek letter Phi, Φ, changes with time, an electromotive force, or Emf, is produced.

The total magnetic flux through a plane area, A, placed in a uniform magnetic field depends on the angle between the direction of the magnetic field and the direction perpendicular to the surface area. The equation is $\mathbf{\Phi = BA\cos(\theta)}$ **.**

The results of Faraday's experiments led to a basic and important law of **electromagnetism** known as **Faraday's Law of Induction**.

This law says that the magnitude of the **Emf, ε , induced in a circuit** equals the time rate of change of the **magnetic flux** through the circuit.

Or we can say, $\varepsilon = N \cdot \Delta\Phi / \Delta t$, with N as the number of turns in the coil.

Since the magnetic flux is the product of the magnetic field, B, the area, A, and the cosine of the angle between the magnetic field and the normal to the surface, there are three possible ways the flux can change with time; the field, B, or the area, A, or the angle theta.

When the magnetic field, **B, changes with time**, the Emf is expressed as:

$\varepsilon = dB/dt(A\cos(\theta))$, where dB/dt is the rate of change of the magnetic field. This kind of Emf is produced in transformers.

When the area, **A, changes with time**, the Emf is expressed as:

$\varepsilon = B(dA/dt)\cos(\theta)$, where dA/dt is the rate of change of the area.

If the area is a rectangle of length, L, and of width, W, and the side, L, is moving with a velocity, v, in the plane of the area, then the $dA/dt = L \times v$, the cross-product, and the Emf is then given by:

$\varepsilon = B(L \times v)$, with v perpendicular to B. This kind of Emf is produced when wires move in magnetic fields.

When the angle, **theta, changes with time**, the Emf is expressed as:

$\varepsilon = BAd(\cos(\theta))/dt$, where $d(\cos(\theta))/dt$ is the rate of change of the cosine function as the angle theta changes.

If $d\theta/dt$ is the rate of change of the angle, in radians/sec, called omega, ω, then the induced Emf is written as:

$\varepsilon = \omega BA\sin(\theta)$. This kind of Emf is produced in electrical generators.

Basically, an Emf can be produced in a closed loop of wire when the wire moves into a magnetic field.

Summing up, an Emf can be induced in the circuit in several ways:

- The magnitude of the magnetic field can change as a function of time.
- The area of the circuit can change with time.
- The direction of the magnetic field relative to the circuit can change with time.
- Any combination of the above can change.

Again, it is important to note that the magnitude of the induced Emf **depends on the rate** at which the magnetic field is changing.

Motional Emf: A potential difference will be maintained across a conductor of length, L, moving with velocity, v, in a magnetic field as long as the direction of motion through the field is not parallel to the field direction. **$\varepsilon = BLv$**

If the motion is reversed, the polarity of the potential difference will also be reversed.

Lenz's Law: The polarity of the induced Emf is such that it tends to produce a current that will create a magnetic flux to oppose the change in flux through the circuit , $\boldsymbol{\varepsilon = -N \cdot \Delta\Phi / \Delta t}$ **.**

Maxwell's equations applied to free space are:

(I) **Gauss's Law:** The total electric flux through any closed surface equals the net charge inside that surface divided by epsilon naught.

$$\int E \cdot dA = Q_{enc}/\varepsilon_o$$

This law describes how charge creates the electric field.

(II) **Gauss's Law for Magnetism:** The net magnetic flux through a closed surface is zero.

$$\Phi_{net} = \int B \cdot dA = 0$$

(III) **Faraday's Law of Induction:** The **line integral** of the electric field around any closed path equals the rate of change of magnetic flux through any surface area bounded by the path.

$$\varepsilon = \int E \cdot ds = -d\Phi_B/dt$$

This law describes how a changing magnetic field creates an electric field, and results in an Emf.

(IV) **Ampere-Maxwell Law:** The line integral of the magnetic field around any closed path is determined by the sum of the net conduction current through that path and the rate of change of electric flux through any surface bounded by that path.

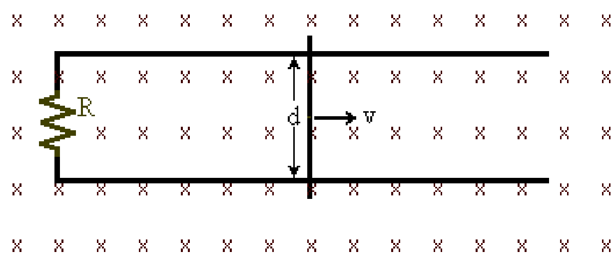
$$\int B \cdot dl = \mu_o(I + d/dt(\varepsilon_o \int E \cdot dA))$$

This law describes how changing electric fields create a magnetic field.

These four equations, together with the **Lorentz Force Law** (based on the cross-product of v and B), $\mathbf{F = qE + q(v \times B)}$, describe all electromagnetic phenomena.

Here are solved problems on Faraday's Law:

1. In the figure below, with R = 6 Ω, d = 1.2 m, a uniform 2.5 T magnetic field is directed into the page. At what speed should the bar be moved across the page to produce a 0.75 A current in the resistor?



Solution: $\varepsilon = IR = Bvd\cdot\cos\theta$ Therefore $v = IR/(Bd)$
$v = 0.75\ A\cdot 6\ \Omega\ /(2.5\ T\cdot 1.2\ m)$ = **1.5 m/s**

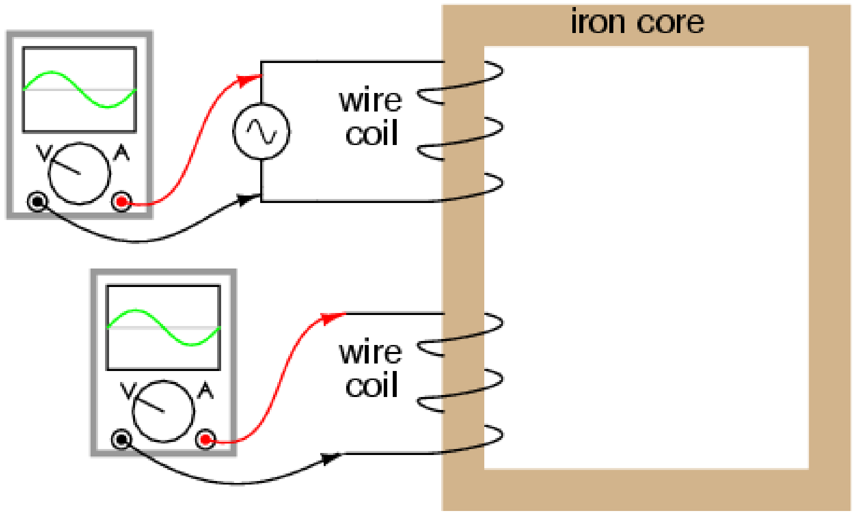
2. A 30-turn square coil (length of side = 12 cm) with a total resistance of 2.5 Ω, is placed in a uniform magnetic field directed perpendicularly to the plane of the coil. The magnitude of the field varies with time according to $B = \beta e^{8t}$, where β = 50 mT and t is measured in seconds. What is the magnitude of the induced emf in the coil at t = 0?

Solution: $\varepsilon = N\cdot d\Phi/dt$ N = 30 turns $\Phi = BA\cos\theta$
Area, $A = .0144\ m^2$ $B = \beta e^{8t}$ β = 50 mT θ = 0
$\varepsilon = N\cdot d(BA\cos\theta)/dt = (NA)dB/dt = 30\cdot .0144\cdot 8\cdot .050$
ε = **.17 V**

3. The induced electric field 12 cm from the axis of a solenoid with 10 cm radius is 45 V/m. Find the rate of change of the solenoid's magnetic field.

Solution: $\varepsilon = \int E\cdot ds = -d\Phi_B/dt$, $2\pi r\cdot E = \pi R^2\cdot dB/dt$
$dB/dt = (2r\cdot E)/R^2 = (2\cdot .12\ m\cdot 45\ V/m)/(.10\ m)^2$
$dB/dt = 1.08\times 10^3$ T/s = **1.08 T/ms**

Chapter 10: Inductance.



The **property of inductance** applies when a wire is wound into a coil it then has a tendency to oppose any change in current.

It can also be said that inductance is the property of a circuit by which energy is stored in the form of an electromagnetic field.

A wire wound into a coil has the ability to produce a counter or **"back" Emf** (opposing current flow) and therefore has a value of inductance.

When the current in a coil changes with time, an Emf is induced in the coil according to Faraday's Law. This Emf (self-induced) is given by $\boldsymbol{\varepsilon_L = -L\,dI/dt}$, with L being the **self inductance** of the coil.

The standard value of inductance is the **Henry (V·s/A)**, a large value which like the Farad for capacitance is rarely encountered in electronics today.

Typical values of units are milli-henries mH, one thousandth of a henry or the micro-henry uH, one millionth of a henry.

A small straight piece of wire exhibits inductance (probably a fraction of a uH) although not of any major significance until we reach UHF frequencies.

The **inductance of any coil** is $\mathbf{L = N\Phi_B/I}$, with Φ_B the magnetic flux and N the number of turns.

The **inductance of an air-core solenoid** is given by, $\mathbf{L = \mu_o N^2 A/L}$, with A being the cross-sectional area and L the length.

The value of an inductance varies in proportion to the number of turns squared.

Having two turns the value would be four units while three turns would produce nine units although the length of the coil also enters into the equation.

Remember that inductance measures the electromagnetic induction of an electric circuit component; it is a **property of the component itself** rather than of the circuit as a whole.

The **self-inductance**, L, of a circuit component determines the magnitude of the electromotive force (emf) induced in it as a result of a given voltage.

Faraday's Law applied to an inductor states that a changing current induces a back Emf that opposes the change. Or $\boldsymbol{\varepsilon} \mathbf{= L(di/dt)}$. Where $\boldsymbol{\varepsilon}$ is the voltage across the inductor and L is the inductance measured in henries (H).

The inductance will tend to smooth sudden changes in current just as the capacitance smoothes sudden changes in voltage. Of course, if the current is constant there will be no induced EMF.

So unlike the capacitor which behaves like an open-circuit in DC circuits, an inductor behaves like a short-circuit in DC circuits.

Applications using inductors are less common than those using capacitors, but inductors are common in high frequency circuits.

Inductors are never pure inductances because there is always some resistance in and some capacitance between the coil windings.

When **choosing an inductor** (occasionally called a choke) for a specific application, it is necessary to consider the following:

- value of the inductance
- DC resistance of the coil
- current-carrying capacity of the coil windings
- breakdown voltage between the coil and the frame
- frequency range in which the coil is designed to operate.

To obtain a **very high inductance** it is necessary to have a coil of many turns. The inductance can be further increased by winding the coil on a closed-loop iron or ferrite core. To obtain as pure an inductance as possible, the DC resistance of the windings should be reduced to a minimum.

This can be done by increasing the wire size, which of course, increases the size of the choke. The size of the wire also determines the current-handling capacity of the choke since the work done in forcing a current through a resistance is converted to heat in the resistance.

Magnetic losses in an iron core also account for some heating, and this heating restricts any choke to a certain safe operating current. The windings of the coil must be insulated from the frame as well as from each other.

Heavier insulation, which necessarily makes the choke more bulky, is used in applications where there will be a high voltage between the frame and the winding.

The losses sustained in the iron core increase as the frequency increases.

Large inductors, rated in henries, are used principally in power applications.

The frequency in these circuits is relatively low, generally 60 Hz or low multiples thereof. In high-frequency circuits, such as those found in FM radios and television sets, very small inductors (of the order of micro-henries) are frequently used.

If a **resistor and an inductor are connected in series** to a battery of Emf, **ε**, and a switch is closed at t = 0, the current in the circuit varies with time according to, $\mathbf{I(t) = \varepsilon/R(1 - e^{-t/\tau})}$. Here τ , tau, is the **time constant** of the **RL circuit**, and $\boldsymbol{\tau = L/R}$.

If the battery is removed from the circuit, the current decays exponentially with time according to the equation $\mathbf{I(t) = (\varepsilon/R)\cdot e^{-t/\tau}}$.

Don't forget, the initial current in the circuit is I_o, given by $\mathbf{I_o = \varepsilon/R}$.

The energy stored in the magnetic field of an inductor carrying a current I is given by $\mathbf{U_B = \frac{1}{2}LI^2}$.

The energy per unit volume, where the magnetic field is B, is given by the equation $\mathbf{u_B = B^2/2\mu_o}$.

In an **LC circuit** with zero resistance, the charge on the capacitor varies with time according to the equation, $\mathbf{Q = Q_{max}\cos(\omega t + \varphi)}$.

The current also varies with time given by the equation, $\mathbf{I = dQ/dt = -\omega Q_{max}\sin(\omega t + \varphi)}$. Here Q_{max} is the maximum charge on the capacitor, φ (Phi) is the phase angle, and **ω** is the angular frequency of the oscillation, and $\boldsymbol{\omega = 2\pi f}$.

It can be shown that **$\omega = 1/\sqrt{(LC)}$** .

The **energy in an LC** circuit continuously transfers energy stored in the capacitor and energy stored in the inductor.

The **total energy of the LC circuit** at any time t is, of course, **$U = U_c + U_L$**. Using substitution we get **$U = (Q^2_{max}/2C)\cos^2(\omega t) + (LI^2_{max}/2)\sin^2(\omega t)$**.

At t = 0, all of the energy is stored in the electric field of the capacitor **$U_c = Q^2_{max}/2C$** . Eventually, through time, all of the energy is transferred to the inductor, **$U_L = LI^2_{max}/2$** .

However, the total energy remains constant because the **losses are neglected** in the ideal LC circuit.

The charge and current in an RLC circuit exhibit a **damped harmonic behavior** for small values of R. This is analogous to the damped harmonic motion of a mass-spring system in which friction is present.

Here are some solved problems on Inductance:

1. A coil has a resistance of 5.0 Ω and an inductance of 100 mH. At a certain instant in time after a battery is connected, the current is 2.0 A, and is increasing at a rate of di/dt = 20 A/s. What is the voltage V of the battery? What is the time-constant of the circuit? What is the final value of the current?

Solution: $V = iR + L\cdot di/dt = 2A\cdot 5.0\Omega + .10H\cdot 20A/s$
V = **12 V** Time constant, $\tau = L/R = .10H/5\Omega =$ **.02 s**
final value of current, $I = V/R = 12V/5\Omega =$ **2.4 A**

2. What is the inductance of a series RL circuit in which R = 1.0 kΩ if the current increases to one-third of its final value in 30 μs?

Solution: $R = 1.0\text{x}10^3\ \Omega$ $I = I_o/3$ $t = 30\text{x}10^{-6}$ s
$I = I_o\cdot(1 - e^{-t/\tau})$ $\tau = R/L$ $I_o/3 = I_o\cdot(1 - e^{-tR/L})$
$2/3 = e^{-tR/L}$ $\ln(2/3) = -(30\text{x}10^{-6}\ \text{s}\cdot 1.0\text{x}10^3\ \Omega)/L$
Therefore, $L = -(.030)/\ln(2/3) =$ **74 mH**

3. A 10 mH inductor is connected in series with a resistor of 10 Ω, a switch, and a 6V battery. What is the time constant of the circuit? How long after the switch is closed will the current reach 99 percent of its final value?

Solution: L = 10 mH R = 10 Ω V = 6 V $\tau = L/R$
τ = 10 mH/10 Ω = **1 ms**

$I = I_o \cdot (1 - e^{-t/\tau})$ $.99 \cdot I_o = I_o \cdot (1 - e^{-t/\tau})$
Therefore, $.99 = 1 - e^{-t/.001s}$ Or, $.01 = e^{-t/.001s}$
$\ln .01 = -t/.001$ s $t = -.001$ s$(\ln .01)$ = **4.6 ms**

3. If one wished to construct a circuit where electric charge originally stored on a capacitor flows through an inductor then back again, what value of inductance should one place in series with a fully-charged 100 μF capacitor to get the circuit to resonate at 60 Hz?

Solution: C = 100 μF f = 60 Hz and $f = \omega/2\pi$
$\omega = 1/\sqrt{(LC)}$ $2\pi f = 1/\sqrt{(LC)}$ $LC = 1/(4\pi^2 f^2)$
Now, $L = 1/(4\pi^2 f^2 C) = 1/(4\pi^2 \cdot 60^2 \cdot 1x10^{-4})$
Therefore, L = **70.3 mH**

4. The magnetic field in a superconducting solenoid is 3.0 T. How much energy is stored in the solenoid, in J/m^3?

Solution: B = 3.0 T, $u_B = B^2/2\mu_o$, $\mu_o = 4\pi x10^{-7}$Tm/A
$u_B = (3.0\ T)^2/(2 \cdot 4\pi x10^{-7}Tm/A)$ = **$3.6x10^6$ J/m^3**

Appendix

List of Physical Constants

Permittivity of free space, $\mu_o = 4\pi x10^{-7}$ Tm/A

Permeability of free space, $\varepsilon_o = 8.85x10^{-12} C^2/Nm^2$

Rest mass of the electron, $m_e = 9.11 \times 10^{-31}$ kg

Rest mass of the proton, $m_p = 1.67 \times 10^{-27}$ kg

Rest mass of the neutron, $m_n = 1.67 \times 10^{-27}$ kg

Charge on the proton, $q_p = 1.602x10^{-19}$ C

Charge on the electron, $q_e = -1.602x10^{-19}$ C

Acceleration due to gravity (sea level), g = 9.8 m/s^2

Prefixes for Powers of 10

Prefix	Symbol	Notation
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}

Units of Physical Quantities

Electric field strength, E:	V/m or N/C
Electric Potential (voltage), V:	J/C or Volt
Electric Current, I:	Ampere, A
Resistance, R:	Ohm, Ω
Capacitance, C:	Farad, F, or Coulomb/Volt
Magnetic Field Strength, B:	Tesla, T or N/Am
Energy density, u:	J/m^3