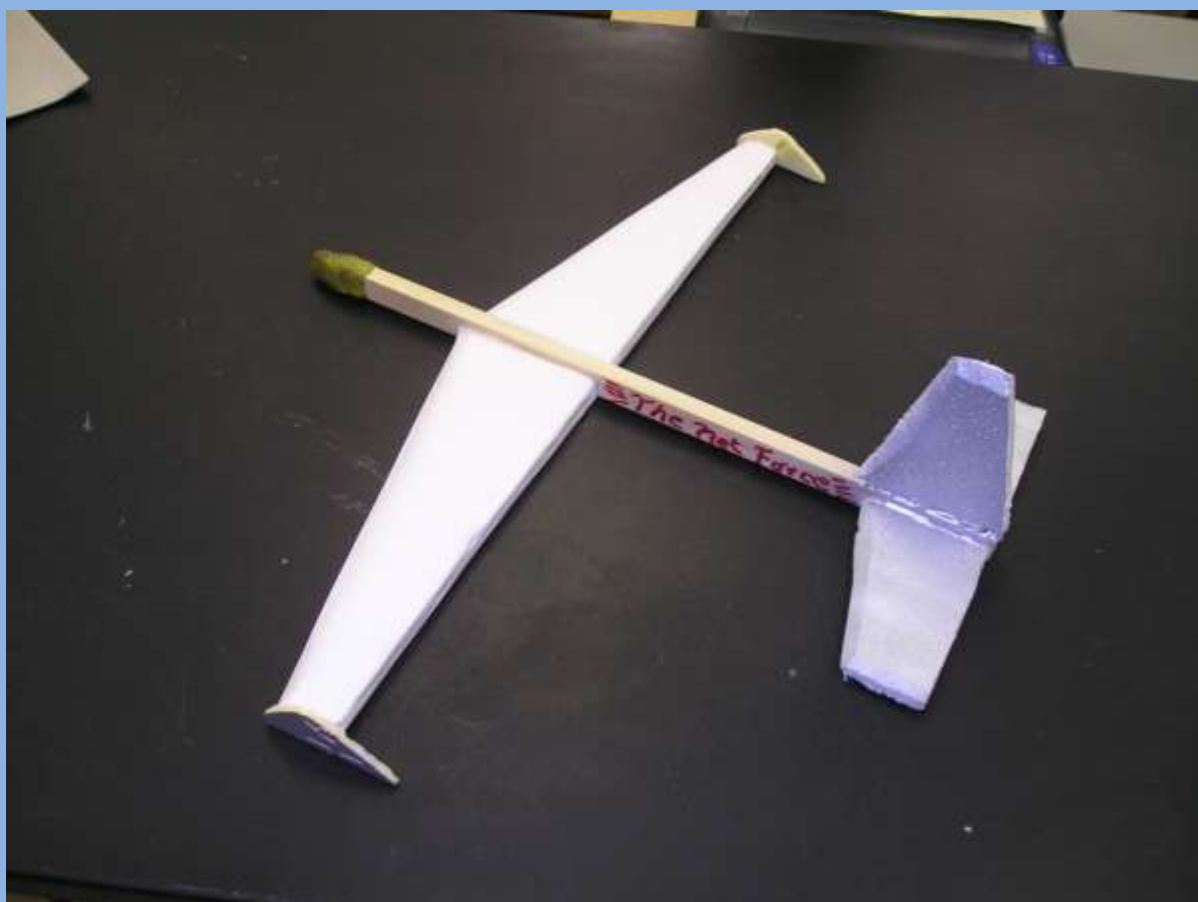
Physics

Simply Put



Volume 1: Mechanics with Calculus

by Dr. Ronald C. Persin

Lnk2Lrn.com

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Simply Put

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Dr. Ronald C. Persin

Founder of Lnk2Lrn.com

"From Newtonian mechanics,

Through Quantum Theory,

Without Physics,

Life would be dreary." RCP

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Abstract

This book is a compilation of Dr. Persin's website notes that were originally published on his website, Lnk2Lrn.com. According to his many physics students over the years, these notes were especially helpful in understanding important concepts in calculus-based physics.

Topics separate chapters on physics and measurement, vectors and scalars, motion in one and two dimensions, forces and Newton's Laws, work and energy, momentum, rotational mechanics, static equilibrium and elasticity, universal gravitation, and oscillatory motion.

Sample problems solved in step-by-step manner appear at the end of each chapter.

It is the intent of the author that this book will help students to be prepared for class on a daily basis with all the knowledge of the day before.

Chapter 1 - Physics and Measurement.

The **Scientific Method** is probably the most efficient problem-solving tool ever devised. There are 6 steps in the Scientific Method:

- Define the problem
- Gather information
- State your hypothesis
- Test the hypothesis
- Form your conclusion
- Publish the results.

The scientific method is the process by which scientists, collectively and over time, endeavor to construct an accurate (that is, reliable, consistent and non-arbitrary) representation of the natural world.

The **Metric System** (*Systeme Internationale*, SI) was first introduced by the French Academy of Science in 1795 as an attempt to unify existing systems. The SI contains the basic units for length(meter, m), mass (kilogram, kg), and time(second, s), with speed (m/s), volume (m^3), and density (kg/m^3) as some derived units.

All calculations must be done observing significant digits and scientific notation. When a number is expressed in **scientific notation**, the number of significant figures is the number of digits needed to express the number to within the uncertainty of measurement.

We always round to the least precise measurement. For example, if you multiply 2.5 cm x 1.23 cm, the result is expressed as 3.1 cm^2 .

The number of **significant figures** of a product or quotient of two or more quantities is equal to the smallest number of significant figures for the quantities involved.

For example, if you multiply 5.2 x 3.751 x 6.43, your answer must be written using only two significant digits as 130.

For addition or subtraction, the number of significant figures is determined with the smallest significant figure of all the quantities involved.

For example, the sum 10.234 + 5.2 + 100.3234 is 115.7574, but should be written 115.8 (with rounding), since the quantity 5.2 is significant only to only one decimal place.

Order of magnitude (power of 10) calculations provide quick estimates for answers to certain questions. An order of magnitude calculation is an *estimate* to determine if a more precise calculation is necessary. We round off or guess at various inputs to obtain a result that is usually reliable to within a factor of 10.

Specifically, to get the order of magnitude of a given quantity, we round off to the closest power of 10 (example: 75 kg is expressed as 10^2 kg). Another example, the average distance from the Earth to the Sun is 93,000,000 miles. In scientific notation this is $9.3x10^7$ miles. But since 9.3 is closest to 10^1, we would express the order of magnitude as 10^8 miles.

A **frame of reference** (inertial) is a coordinate system for specifying the precise location of objects in space. It is assumed to be non-accelerating so that all three of Newton's laws are valid.

Accuracy refers to the agreement of a measured value with an accepted value. **Percent error** measures accuracy.

There are occasions when an error in measurement is due to **parallax** which is the apparent shift in position of an object when it is viewed from different angles.

Precision is the agreement of a set of measured values with each other and is determined by the fineness of divisions on a scale. It can be measured by **average deviation**.

All graphs are plotted with the **independent (control) variable** on the x-axis, and the **dependent (measured) variable** on the y-axis.

Graphs can show direct (linear), inverse (hyperbolic), periodic (sinusoidal), quadratic (parabolic), or chaotic relationships.

All equations must be **dimensionally correct**. We use dimensional analysis (factor labeling) to determine if equations are correct.

The **density of a substance** is defined as its mass per unit volume. We use the Greek letter rho, ρ, for density and the equation is $\rho = m / V$.

Some equations that you may remember from mathematics that are also important in physics are: $A = \pi r^2$, $C = 2\pi r$, $A = 4\pi r^2$, $V = \pi r^2 h$, $V = 4/3\ \pi r^3$, and $d = vt$.

Realize that variables in Physics are case sensitive. For example, **A** is area, but **a** is acceleration. Another example, **T** is temperature, but **t** is time.

We need to remember **SOHCAHTOA** and the **Pythagorean Theorem** in order to compute the value of unknown sides and angles of right triangles when dealing with vector quantities.

We also need these **steps to solve any problem in Physics**:

(1) read the problem and identify the given variables

(2) determine what you are asked to solve for

(3) find the correct formula to use

(4) isolate the unknown

(5) substitute-in the given information and simplify.

Here is an example of how to use the steps to solve a problem:

If a 2.5 cm^3 volume of liquid has a density of 3.0 g/cm^3, what is the mass?

Solution:

(1) $V = 2.5\ cm^3$ $\rho = 3.0\ g/cm^3$ (2) $m =$ ____

(3) $\rho = m / V$

(4) $m = \rho \cdot V$

(5) $m = (3.0\ g/cm^3)\cdot(2.5\ cm^3) =$ **7.5 g**

Chapter 2 - Motion in One-Dimension.

In Physics, **Mechanics** is the study of motion. This is divided into **Kinematics**, the study of HOW things move, and **Dynamics**, which is concerned with WHY they move.

Galileo (1564-1642) was the first to study motion and developed Kinematics. He performed his experiments by dropping objects from the Leaning Tower of Pisa, and/or rolling spheres along level surfaces and down ramps.

Isaac Newton (1642-1727), a theoretical physicist, formulated Dynamics by deriving his 3 Laws of Motion and the Law of Universal Gravitation. We will study his work in depth when we get to Chapter 4.

Galileo's study of motion produced the **motion formulas**:

- $\Delta x = v_{avg} \cdot \Delta t$, with $v_{avg} = (v_o + v)/2$
- $v = v_o + a \cdot \Delta t$
- $v^2 = v_o^2 + 2a \cdot \Delta x$
- $\Delta x = v_o \cdot \Delta t + \frac{1}{2} a \cdot (\Delta t)^2$

For falling objects, the **acceleration due to gravity** is 9.8 m/s^2 at sea level.

Instantaneous velocity can be computed by finding the slope of a tangent line at any point on the graph of displacement as a function of time.

Instantaneous acceleration can be computed by finding the slope of a tangent line at any point on the graph of velocity as a function of time.

Instantaneous velocity and acceleration can also be evaluated using another method involving limits and the **first derivative in Calculus**, where, **v = dx/dt** and **a = dv/dt**.

Integral calculus can be used to derive velocity from acceleration, and displacement from velocity. In terms of Calculus, **v = ∫a dt + C**.

Similarly, **x = ∫v dt + C**.

Displacement, velocity, and acceleration can all be demonstrated **graphically**. For example, straight lines on a position-time graph indicate constant velocity, determined by the slope.

The **area under a velocity-time graph** is displacement while the slope of the graph indicates acceleration. The area under an acceleration-time graph is the velocity.

Here are some solved example problems in one-dimensional motion:

(1) An object falls 200.0 from rest. Find its velocity when striking the ground.

Solution:

$v_o = 0$ $a = g = 9.8\ m/s^2$ $\Delta x = 200.0\ m$

$v =$ _____

$v^2 = v_o^2 + 2a \cdot \Delta x$

$v^2 = 0^2 + 2(9.8\ m/s^2) \cdot 200.0\ m$

$v = \sqrt{(3920)}\ m/s =$ **62.6 m/s**

(2) The position of a particle moving along the x axis is given by, $x = 12t^2 - 2t^3$, where x is in meters and t is in seconds. What is the maximum velocity reached by the particle and when is it reached?

Solution:

$x = 12t^2 - 2t^3$ $dx/dt = v = 24t - 6t^2$

$dv/dt = a = 24 - 12t$

maximum velocity occurs when the acceleration equals 0.

$24 - 12t = 0$ t = **2 sec.**, when maximum velocity is reached.

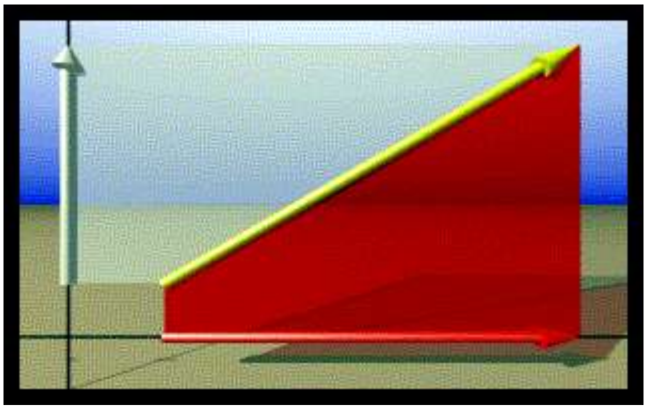
$v = 24t - 6t^2 = 24(2) - 6(2^2) =$ **24 m/s**

Chapter 3 – Vectors and Scalars.

Vectors

Vectors are used to describe multi-dimensional quantities. Multi-dimensional quantities are those which require more than one number to completely describe them. Vectors, unlike scalars, have two characteristics, **magnitude and direction**.

A vector is indicated by an uppercase letter either in boldface or with an arrow over the top. For example, **A** or **Â**. In a diagram, vectors are indicated by arrows.



Examples of vector quantities for now are: position in a plane, position in space, displacement, velocity, acceleration, and force.

Scalars

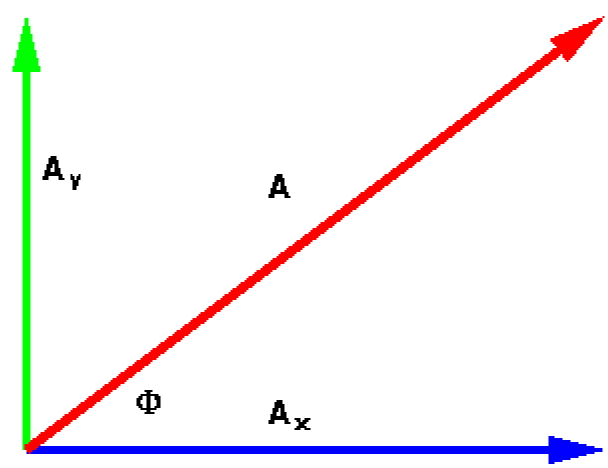
Scalars are used to describe one- dimensional quantities, that is, quantities which require only one number to completely describe them. They have **magnitude only**. Direction does not apply.

There are cases where scalars can be combined mathematically, but we will save that for later. Some examples of scalar quantities are: speed, distance, temperature, mass, time, volume, density, length, area, and energy.

Vector Diagrams

Any vector can be resolved into **perpendicular component vectors** using sine and cosine functions. All you have to do is diagram the vector from the origin in the x-y plane. Then draw a rectangle around it using the original vector as the diagonal of the rectangle.

For vector problems just remember SOHCAHTOA.



Thus, $A_x = A \cdot \cos(\Phi)$ and $A_y = A \cdot \sin(\Phi)$. The magnitude of **A** = $|\mathbf{A}| = \sqrt{(A_x^2 + A_y^2)}$. The direction of **A** is given by $\Phi = \tan^{-1}(A_y/A_x)$.

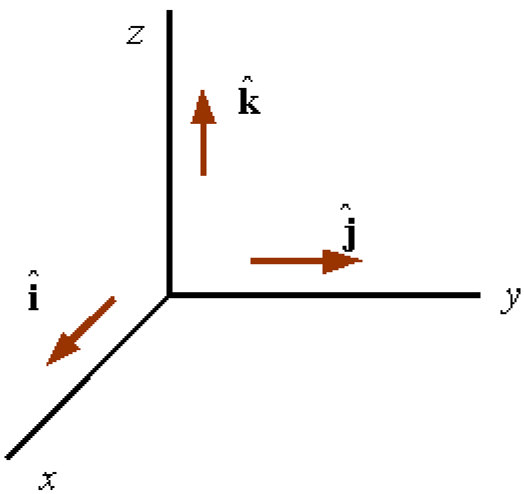
Some Properties of Vectors

Two vectors are equal only if they have the same magnitude and direction.

To find the opposite of a given vector just keep the same magnitude but point it in the opposite direction. ex. **A** - **B** = **A** + (**-B**)

Vectors can also be expressed using **polar coordinates (r , θ)** specifying the length of the radius vector **r** , and the angle of rotation, **θ**, from the positive x-axis.

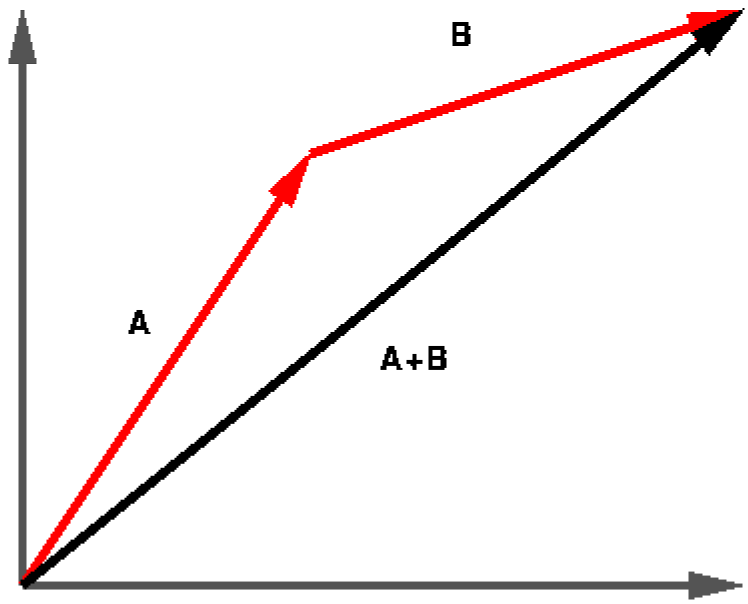
Additionally, in a two-dimensional coordinate system, vectors can be denoted using the unit vectors **î** and **ĵ**. Each unit vector has magnitude equal to 1, and they point in the x and y directions, respectively. We can easily add the third dimension, or z direction using unit vector **k**.



Addition of Vectors

Vectors can be added graphically using the head-to-tail method. You begin by drawing the first vector in a coordinate system, and then drawing the second vector from the endpoint of the first, and so on.

Then you draw a single vector from the origin to the head of the last vector.

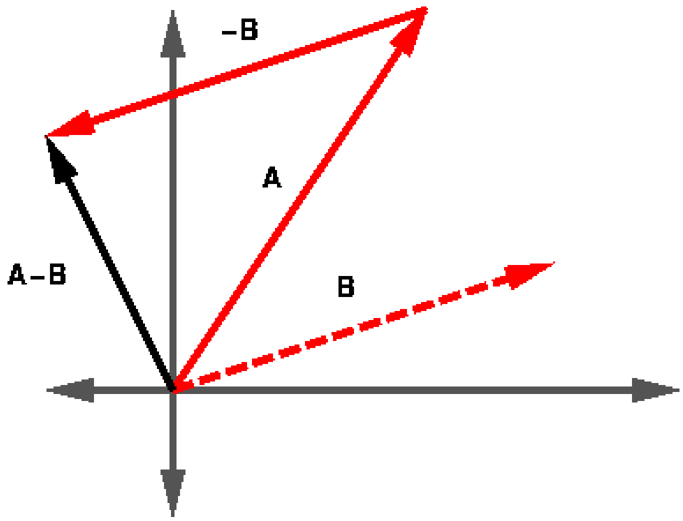


Vector Subtraction

The vector difference works the same as vector addition except that we multiply the vector we are subtracting by **-1**. It is much like subtracting two numbers: **A - B = A + (-B)**.

The diagram on the next page illustrates vector subtraction in the tip-to-tail style.

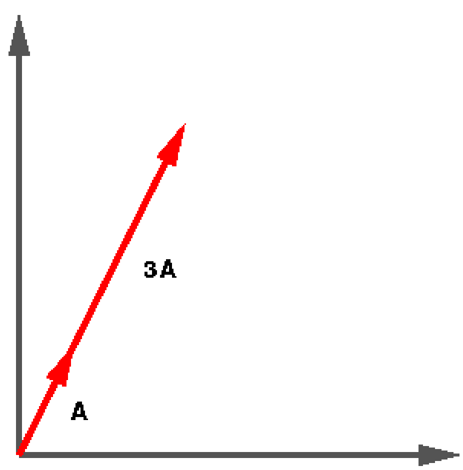
The original **B** vector is shown as a dashed arrow.



Multiplication of a Vector by a Scalar

A vector may be multiplied by a scalar by multiplying each of its components by that number. Notice that the vector does not change direction, only length.

If A = (1,2) then 3A = (3,6). This is shown below.



Here are some sample vector problems:

1. If **A** = [15, 80°] and **B** = 12**i** – 16**j**, what is the magnitude of **A** – **B**?

Solution:

A_x = A·cos(θ) = 18·cos(80) = 3.13

A_y = A·sin(θ) = 18·sin(80) = 17.7

Therefore **A** = 3.13**i** + 17.7**j**

A – **B** = (3.13 – 12)**i** + (17.7 + 16)**j** = -8.87**i** + 33.7**j**

For magnitude, │ **A – B** │ = $\sqrt{(-8.87^2 + 33.7^2)}$ = **34.8**

2. If **A** = 12**i** – 16**j** and **B** = –24**i** + 10**j**, what is the direction of the vector **C** = 2**A** – **B**?

Solution:

2**A** – **B** = 2**A** + (– **B)**   2**A** = 24**i** – 32**j**

-**B** = –(-24**i** + 10**j**) = 24**i** - 10**j**

C = 2**A** + (-**B)** = (24 + 24)**i** + (-32 – 10)**j** = 48**i** – 42**j**

For the direction of **C**:

θ = $\tan^{-1}(C_y/C_x)$ = $\tan^{-1}(-42/48)$ = $\tan^{-1}(-.875)$ = **-41°**

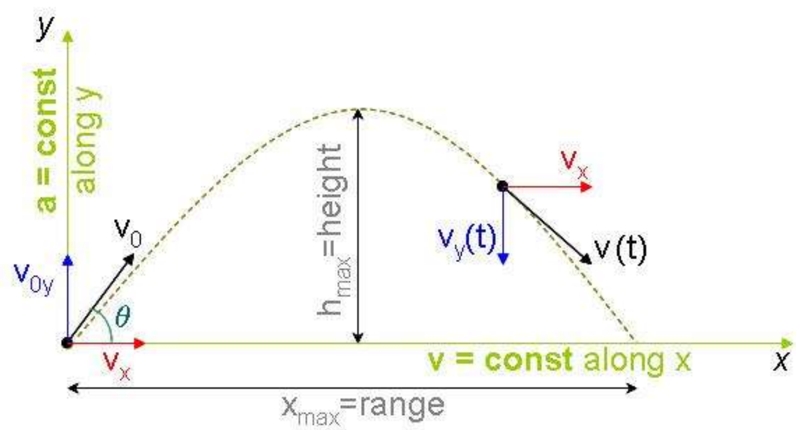
Chapter 4 - Motion in Two Dimensions.

In two-dimensional motion, the **horizontal and vertical components** of the motion must be regarded independently. For these two directions we use x and y, respectively.

For example, if an object is **projected from the ground** with an initial velocity v_i at an **angle of elevation θ_o**, then we can use SOHCAHTOA to find out how fast it is moving in the x and y directions.

An object launched from the ground at some angle **θ_i** is called a **projectile**.

The path it travels is an **inverted parabola** called its **trajectory**. A classic example would be the motion of golf ball when struck with a club. Can you think of a few more?



The initial velocity in the x direction is given by the equation, $\mathbf{v_x = v_{ox} = v_o \cdot cos(\theta_o)}$. The velocity of the object in the y direction is $\mathbf{v_{oy} = v_o \cdot sin(\theta_o)}$.

The acceleration is that of gravity which acts only in the y direction. It is given by $\mathbf{a_y = g = 9.8 m/s^2}$.

The acceleration in the x direction is $\mathbf{a_x = 0}$.

We still have the **motion formulas** from the study of kinematics developed by Galileo.

We still know them as:

- $\Delta x = v_{avg} \cdot \Delta t$, with $v_{avg} = (v_o + v)/2$
- $v = v_o + a \cdot \Delta t$
- $v^2 = v_o^2 + 2a \cdot \Delta x$
- $\Delta x = v_o \cdot t + ½a \cdot (\Delta t)^2$

The task now is to adjust these for the separate x and y directions.

Doing this, we get the following set of kinematics equations to analyze the motion of a projectile launched at an angle. For the x direction we only have $\Delta x = v_{ox} \cdot \Delta t$.

For the y direction we have 3 equations:

- $v_y = v_{oy} + g \cdot \Delta t$,
- $v_y^2 = v_{oy}^2 + 2g \cdot \Delta y$
- $\Delta y = v_{oy} \cdot \Delta t + ½g \cdot (\Delta t)^2$

In the **absence of air resistance** a projectile has a constant horizontal velocity and a constant downward free-fall acceleration which effects the vertical velocity, subtracting 9.8m/s from it on the way up, and the way down.

Frame of Reference

A **frame of reference** must be used when analyzing the **relative motion** of two objects. When two observers are moving relative to each other there would not be agreement on the displacements and velocities of an object in motion when each is using his/her own frame of reference.

For example, a person standing in a moving subway car, and facing towards the back of the car, drops a book. According to the frame of reference of the person in the car, the book fell in a straight line to the floor.

An observer standing outside on the subway platform as the car goes by, sees the book traveling in a parabolic path toward the floor.

Therefore, the motion of an object depends on your frame of reference. This is also occurs when boats travel in moving streams and when planes encounter moving air masses. Also, sometimes you hear about certain records in track and field that are not allowed if it is determined that athletes were "wind aided."

Another type of two-dimensional motion is **periodic motion** in which an object moves back and forth over the same path. An example of this would be the motion of the **simple pendulum**.

Also included is **uniform circular motion** in which an object has a constant speed and is accelerated toward the center of the circular path.

This introduces the concept of a **centripetal (center seeking) acceleration**, given by $\mathbf{a_c = v^2/r}$, which can be derived using geometry and the properties of vectors.

Also, if we need the time for one revolution, called the **period, T** , we have $\mathbf{T = 2\pi r / v}$.

If a **particle moves along a curved path** in such a way that the magnitude and direction of v change with time, the particle has an **acceleration vector** that can be described with **two component vectors**.

The **radial component vector** arises from the change in direction of v, which is the centripetal acceleration, $\mathbf{a_c = v^2/r}$.

The **tangential component vector** is based on the change in magnitude of v, and is found with the derivative $\mathbf{a_t = dv/dt}$.

The **total or net acceleration** can be found with the vector sum of these two accelerations which occur at right angles, so we use the Pythagorean Theorem and inverse tangent function.

Here is a sample projectile motion problem:

A rock is thrown from the top of a building with an initial velocity in the x direction of 30 m/s. If the top of the building is 30 m above the ground, how fast will the projectile be moving just before it strikes the ground?

Solution: $v_{ox} = 30$ m/s $a_x = 0$ $v_{oy} = 0$

$a_y = g = 9.8\ m/s^2$ $\Delta y = 30$ m $v_y =$ ____

$v_y^2 = v_{oy}^2 + 2g\cdot\Delta y = 0^2 + 2(9.8\ m/s^2)(30\ m)$

$v_y = \sqrt{(588)}$ m/s = 24.2 m/s

$v = \sqrt{(v_x^2 + v_y^2)} = \sqrt{(30^2 + 24.2^2)} =$ **38.5 m/s**

Chapter 5 - Forces and Newton's Laws.

A **force** is a push or a pull on an object. Forces can act through physical contact (**contact forces**) or at a distance (**field forces**). All forces are vectors because they have both magnitude and direction. A **free-body diagram** shows force vectors as arrows.

The **unit of force** in MKS is the Newton, named after **Isaac Newton** (1642 – 1727).

A Newton is another name for a kg m/s^2. In CGS we use the Dyne as a unit of force.

The **four fundamental forces** are gravity, electromagnetic, strong nuclear, and weak nuclear.

Isaac Newton determined that the cause of motion is force. This study is known as Dynamics.

Newton summarized all motion with his three laws.
Law I: An object will remain at rest or in a state of constant motion if the forces acting on it are balanced. This is known as the **Law of Inertia**.

Law II: The acceleration of an object is directly proportional to, and in the direction of, the net force, but varies inversely with the mass.

From this law we get the equation that $\mathbf{F_{NET} = ma}$. The net force is the vector sum of all forces acting on an object.

Law III: For every action force there is always an equal and opposite reaction force.

We can now state the difference between **mass and weight**. Mass is the measure of the amount of matter in an object. Weight is the force of gravity on the object. The MKS unit of mass is the kilogram (kg), while the unit of weight is the Newton (N). In CGS we use the gram (g) and the Dyne (Dyn).

To change mass to weight, we use the equation (based on Newton's 2^{nd} Law) $\mathbf{F_g = mg}$. This is the same as F = ma , with $\mathbf{g = 9.8\ m/s^2}$.

There are two kinds of mass, **gravitational and inertial mass**. They are numerically equal but are determined in two different ways.

Friction is a force that opposes the motion of an object. It is a type of electromagnetic force and is caused by surface conditions and the weight of the object being moved. The **coefficient of friction** is given by the Greek letter mu, μ .

The force of friction is determined by multiplying the **coefficient of friction** and the **normal force**, that is, $\mathbf{F_f = \mu F_N}$. The normal force is the contact force of one surface on another. Mathematically, "normal" means perpendicular.

Static friction is usually greater than kinetic friction.

Newton's second law applied to a particle moving in **uniform circular motion** states that the net force must be toward the center.

Uniform circular motion occurs when an acceleration of constant magnitude is perpendicular to the **tangential velocity** and the object maintains a constant speed but is accelerated toward the center of the circle.

This introduces the concept of **centripetal acceleration**, $\mathbf{a_C = v^2/r}$, and, by Newton's second law, **centripetal force**, $\mathbf{F_C = mv^2/r}$.

The central force acting on an object that provides the centripetal acceleration could be have its origin in the following:

- the force of gravity (as in satellite motion)
- the force of friction (as in a car rounding a curve)
- a force exerted by a string (motion in a horizontal circle)

In the case of **motion in a vertical circle**, the force of gravity provides the tangential acceleration and part or all of the centripetal acceleration. The velocity at the bottom of the circle must be at least large enough to provide for a complete round trip. Setting $mg = mv^2/r$, we get $\mathbf{v_{crit} = \sqrt{(r \cdot g)}}$.

For the **conical pendulum**, the horizontal component of the tension in the string provides the centripetal force. Here, $\mathbf{v = \sqrt{(r \cdot g \cdot \tan(\theta))}}$.

In the case of a car rounding an **unbanked curve**, the force of static friction is the central force. This allows us to derive the equation, $\mathbf{v_{crit} = \sqrt{(\mu \cdot r \cdot g)}}$.

When the **curved roadway is banked** at an angle θ, then the horizontal component of the normal force is centripetal. Here, **$v = \sqrt{(r \cdot g \cdot \tan(\theta))}$** .

If a particle moves along a curved path in such a way that the **magnitude and direction of v change with time**, the particle has an acceleration vector that can be described with two component vectors.

The **radial component vector arises** from the change in direction of v , which is the centripetal acceleration, **$a_C = v^2/r$** .

The tangential component vector is based on the change in magnitude of v , and is usually found with the derivative dv/dt .

The **total acceleration** can be found with the vector sum of these two accelerations which occur at right angles, so we use the Pythagorean Theorem and inverse tangent.

Here is a problem that applies Newton's Laws:

What is the magnitude of the *total* force on a driver by the dragster he operates as it accelerates horizontally along a straight line from rest to a value of 60 m/s in 8.0 s? (mass of driver = 80 kg)

Solution:

$v_o = 0$ $v = 60$ m/s $\Delta t = 8.0$ s $m = 80$ kg
F_{total} = ______ $F_{total} = \sqrt{(F_x^2 + F_y^2)}$ $F_y = F_g = mg =$ 80 kg(9.8 m/s^2) = 784 N

$v = v_o + a \cdot \Delta t$ $a = v/\Delta t = (60 \text{ m/s})/(8 \text{ s}) = 7.5 \text{ m/s}^2$

$F_x = ma = 80 \text{ kg}(7.5 \text{ m/s}^2) = 600$ N

$F_{total} = \sqrt{(600^2 + 784^2)}$ = **987 N**

Chapter 6 – Work and Energy.

Work is done in physics when af orce is applied to an object and it undergoes a displacement in thedirection of the force.

Therefore,**Work = Force x Displacement** or

$W = F \cdot \Delta x$. Work is a scalar quantity, even though Displacement and Force are vectors, and can be computed as a **Dot-Product of the two vectors**.

Work is measured in Newton-meters, Nm, or Joules. This unit was named after **James Prescott Joule** (1818-1889), the Scottish physicist who figured- ut the mechanical equivalent of heat.

We usually do work against friction, **$W = \mu mg$**, when sliding an object, and against gravity, **$W = mg\Delta y$**, when lifting an object.

When applying a force at an angle, we use the cosine of the angle to compute the amount of work done, $\mathbf{W = (F\cos\theta)\Delta x}$.

Energy is the ability to do work, and there are two kinds of energy, kinetic and potential. Kinetic energy is due to mass and velocity, $\mathbf{K = \frac{1}{2}mv^2}$.

Gravitational Potential energy is due to an objects position, $\mathbf{U_{GRAV} = mg\Delta y}$.

Potential energy can also be elastic, $\mathbf{U_{ELAS} = \frac{1}{2}kx^2}$, with k being the elastic constant.

Work can also be computed by finding the area under a Force vs Displacement curve. This also means that we can use Integral Calculus to calculate Work, $\mathbf{W = \int F(x)\,dx}$, from x_1 to x_2.

Work can also be explained as the transfer of energy by mechanical means. Mechanical energy is the total kinetic and potential energy present in a given system.

There is a conservation law for energy which states that "energy can change in form but can never be created or destroyed", or $\mathbf{K_i + U_i = K_f + U_f}$.

Neglecting friction, mechanical energy is conserved, so that the total amount remains constant.

The net work done on or by an object is equal to the change in the kinetic energy of that object. This means that $\mathbf{W = \Delta K = K_f - K_i}$.

Power is the time rate of doing work, with $\mathbf{P = W/t}$, or $\mathbf{P = F\cdot\Delta x/t}$, or even $\mathbf{P = F\cdot v_{avg}}$, and is measured in Watts.

The Watt was named after **James Watt** (1736-1819) from Scotland, who perfected the steam engine and made it practical to use.

Power is also the rate at which energy is transferred, with 1000 watts being, of course, a kilowatt, kw.

Machines with the same power ratings in watts do the same amount of work in different time intervals.

There are **six simple machines:**

- **Pulley**
- **Inclined plane**
- **Wheel and axle**
- **Jackscrew**
- **Lever**
- **Wedge**

For each we can compute Mechanical Advantage, Ideal Mechanical Advantage, Work Output, Work Input, and an Efficiency. All that is needed to know is the amount of work that the machine does, and the amount of work that must be input to operate it.

Here are problems involving work and energy:

1. A large box is pulled across the floor with a rope at a 56.0^{o} angle using a force of 462 N. How much work is done if the box is moved 24.5 m?

Solution:

$\theta = 56.0^{o}$ F = 462 N $\Delta x = 24.5$ m W = _____

$W = (F\cos\theta)\cdot\Delta x = (462\text{ N})(\cos 56^{o})(24.5\text{ m}) =$ **6300 J**

2. A force acting on an object moving along the *x* axis is given by the equation $F(x) = (14x - 3.0x^2)$ N, where *x* is in m. How much work is done by this force as the object moves from $x = -1$ m to $x = +2$ m?

Solution:

Work, $W = \int F(x)\,dx$, evaluated from x_1 to x_2.

$W = \int(14x - 3.0x^2)\,dx$, from $x_1 = -1$ to $x_2 = 2$

$W = (7x^2 - x^3)$, from $x_1 = -1$ to $x_2 = 2$

$W = 7(2^2) - (2^3) - (7(-1)^2 - (-1)^3) = 28 - 8 - (7 + 1) =$ **12 J**

Chapter 7 - Potential Energy.

Potential energy is associated with the position or configuration of an object.

Potential energy can be thought of as stored energy that can be converted to kinetic energy or other forms.

Also, the energy associated with the motion of an object is kinetic energy, $\mathbf{K = \frac{1}{2}mv^2}$.

In working problems involving gravitational potential energy, it is always necessary to set the gravitational potential energy equal to zero at some location. The choice of the zero level is arbitrary, because the important quantity is the difference in potential energy.

It is often convenient to use the surface of the Earth as the zero potential level, or some other level relevant to a particular problem.

The gravitational potential energy of a particle of mass m that is elevated a distance y near the Earth's is $\mathbf{U = mgy}$. This is the same as the amount of work done in lifting the object.

The elastic potential energy stored in a spring of force constant k is **$U = ½kx^2$** . This is also the same amount of work done in compressing the spring a distance x .

A force is conservative if the work it does on a particle is independent of the path the particle takes between the two points. This means that it doesn't matter how you lift an object.

Another way to say this is, a force is conservative if the work it does is zero when the particle moves through an arbitrary closed path and returns to its original position. So if you lift an object and put it back down, then no work is done.

A force that does not meet any of the above described criteria is non-conservative, or sometimes called, dissipative. A force of this type is friction.

A potential energy function U can be associated only with a conservative force. If a conservative force F acts on a particle that moves along the x axis from x_1 to x_2 , the change in the potential energy of the particle equals the negative of the work done by the force.

Mathematically, this means that the change in the potential energy would be the negative of the integral, or **$\Delta U = -\int F(x)\,dx$ from x_1 to x_2**.

The total mechanical energy of a system is defined as the sum of the kinetic energy and potential energy or, **$E = K + U$**.

If no external forces do work on a system, and there are no non- conservative forces, the total mechanical energy of the system is held constant or , **$K_i + U_i = K_f + U_f$** . This is a statement of the **Law of Conservation of Energy**.

The change in total mechanical energy of a system equals the change in the kinetic energy due to internal non-conservative forces plus the change in kinetic energy due to all external forces.

Internal or external forces can either increase or decrease the kinetic energy of a system.

According to the **Work-Energy Theorem**, to change the kinetic energy of a system, work must be done on it, i.e., $\mathbf{W = \Delta K}$.

Here is a solved problem using Conservation of Energy:

> A 10 kg object is dropped from rest. After falling a distance of 50 m, it has a speed of 26 m/s. How much work is done by the dissipative (air) resistive force on the object during this descent?

Solution:

$m = 10$ kg $v_o = 0$ $\Delta y = 50$ m $v = 26$ m/s

$h = 50$ m W_f = ____

$K_i + U_i = K_f + U_f - W_f$ with zero-potential at 50 m below original position

$mgh = ½mv^2 - W_f$

$W_f = ½mv^2 – mgh = m(½v^2 – gh)$

$W_f = 10(½(26^2) – 9.8·50) =$ **-2020 J**

Chapter 8 - Momentum and Collisions.

The momentum of an object is the product of the object's mass and its velocity. We use lowercase p for momentum and this gives the formula **p = mv** .

Since mass is a scalar and velocity is a vector, **momentum is a vector** because the product of a scalar and a vector is a vector. So, anything we

have done with vectors previously can be done with momentum. It can be broken into horizontal and vertical components, and several momentum vectors can produce a single resultant.

The Pythagorean theorem and SOHCAHTOA are used to solve momentum problems.

The unit for momentum in MKS is the kg·m/s , but when working in CGS we use g·cm/s.

The change in momentum, **Δp** , of an object is equal to the impulse that acts that acts on it. This is known as the **Impulse-Momentum Relation**.

This relation can also be derived from Newton's 2nd Law, and the definition of acceleration. Use **F = ma** with **a = Δv/Δt** .

Therefore, impulse equals change in momentum, **F·Δt =m·Δv**. Impulse is the product of the force acting on an object and the time in which it acts.

The formula for impulse is **J = F·Δt**. The unit of impulse in MKS is the Newton-second or N·s.

Impulse can also be derived using Integral Calculus with **J = ∫F(t)dt = Δp**, same as area under a curve.

The **Law of Conservation of Momentum states** that "In a closed, isolated system, the total momentum of the system does not change".

This law means that in all interactions between isolated objects, momentum is always conserved, or, $\mathbf{p_i = p_f}$.

Objects can transfer momentum during collisions, but the total momentum in the system before the collision must equal the total momentum after the collision, again, $\mathbf{p_i = p_f}$.

During a collision, the change in momentum of the first object is equal to and opposite the change in momentum of the second object.

Collisions can be classified as elastic or inelastic based on whether or not energy is also conserved.

If the collision is elastic we have the equation that $\mathbf{m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'}$.

For the inelastic the two objects stick together giving us $\mathbf{m_1v_1 + m_2v_2 = (m_1 + m_2)v_{1,2}}$.

In a **perfectly elastic collision**, both the momentum and kinetic energy are conserved, and therefore $\mathbf{p_i = p_f}$ and $\mathbf{K_i = K_f}$

In a **perfectly inelastic collision**, two objects stick together and move as one mass after the collision. This means that momentum is conserved but kinetic energy is not conserved. $\mathbf{p_i = p_f}$ and $\mathbf{K_i > K_f}$

In an inelastic collision, kinetic energy is converted into internal elastic potential energy when the objects deform. Some kinetic energy is also converted to sound and thermal energy.

Realize that in the macro world,few collisions are perfectly elastic or perfectly inelastic.

There is a special case called **recoil** in which, for example, two objects are pushed against opposite ends of a spring. Clearly the momentum before they are released is zero, so according to the conservation law the momentum after they are released also must be zero.

Recoil is given by the equation $\mathbf{0 = m_1v_1 + m_2v_2}$, which requires that $\mathbf{v_1}$ and $\mathbf{v_2}$ must have opposite directions.

An interesting aspect of the relationship between momentum and kinetic energy is that if the mass, m, and the velocity, v, of an object is known, then we automatically know its momentum and kinetic energy.

Recall that $\mathbf{p = mv}$ and $\mathbf{K = ½mv^2}$, therefore we can express an object's kinetic energy as $\mathbf{K = ½pv}$.

Here is a solved sample momentum problem:
A 3.00 kg object moving East at 5.00 m/s collides with a 2.00 kg object moving North at 10.0 m/s. The objects become entangled and move off as one after the collision. What is the magnitude of the velocity of the entangled mass? What is the direction relative to due East?

Solution: Since momentum is a vector and the objects are moving at 90^o to each other, use the Pythagorean Theorem and the Conservation Law.

$p_f = \sqrt{(3kg \cdot 5m/s)^2 + (2kg \cdot 10m/s)^2} = 25\ kg \cdot m/s$

Divide by total mass, $v_f = p_f/m = (25\ kg \cdot m/s)/(5\ kg) =$ **5 m/s**

To get the direction, use the inverse tangent,

$\theta = \tan^{-1}(p_y/p_x) = \tan^{-1}(20/15) =$ $\mathbf{53^o}$

Chapter 9 - Rotational Mechanics.

When an object spins about an axis, it is said to undergo rotary motion. The axis of rotation is the line about which the rotation occurs. For example, a wheel rotates on its axis.

Circular motion occurs when the entire object revolves around a single point. We say that the earth revolves around the Sun. To find the velocity

we can use $\mathbf{v = 2\pi r/T}$, with T being the rotational period, or time for one complete round trip.

Additionally, any point on a rotating object is moving in a circular path, demonstrating circular motion with a velocity which therefore must be tangent to the circle.

It is not practical to use linear motion quantities to analyze rotary or circular motion, so we use the rotational quantities **angular displacement**, **angular velocity**, and **angular acceleration**.

Angular displacement is given by the Greek letter, theta (θ), and measured in radians (rad).

A radian is the measure of a central angle which intercepts an arc equal in length to the radius of the circle.

For angular velocity we use omega (ω) which then would be measured in rad/sec., and angular acceleration, alpha (α), is in rad/s^2 .

Remember, to convert degrees to radians, use the conversion factor that $\mathbf{180^o = \pi\ rad}$.

The same five linear motion equations are then transformed into the rotational motion equations using these new quantities.

These now become:

- $\mathbf{\Delta\theta = \omega_{avg}\cdot\Delta t}$, with $\mathbf{\omega_{avg}= (\omega_o+\omega)/2}$
- $\mathbf{\omega = \omega_0+\alpha\cdot\Delta t}$,
- $\mathbf{\omega^2 = \omega_o{}^2+2\alpha\cdot\Delta\theta}$
- $\mathbf{\Delta\theta = \omega_o\cdot\Delta t + \frac{1}{2}\alpha\cdot(\Delta t)^2}$.

Uniform circular motion occurs when an acceleration of constant magnitude is perpendicular to the tangential velocity and the object maintains a constant speed but is accelerated toward the center of the circle. This introduces the concept of centripetal (center seeking) acceleration, $\mathbf{a_c = v^2/r}$.

If a particle moves along a curved path in such a way that the magnitude and direction of v change with time, the particle has an acceleration vector that can be described with two component vectors.

The radial component vector arises from the change in direction of v , which is the centripetal acceleration, $\mathbf{a_c = v^2/r}$, and the tangential component vector $\mathbf{a_t = \Delta v/\Delta t}$, is based on the change in magnitude of v .

The total acceleration can be found with the vector sum of these two accelerations which occur at right angles, so we use the Pythagorean Theorem and get $\mathbf{a_{total} = \sqrt{(a_c^2 + a_t^2)}}$, and $\mathbf{\theta = \tan^{-1}(a_c / a_t)}$.

We have equations which relate angular and linear quantities. For example, for arc length, $\mathbf{s = r \cdot \Delta\theta}$; for linear velocity, we have, $\mathbf{v = \omega \cdot r}$, and acceleration, $\mathbf{a = \alpha \cdot r}$.

If force applied to an object perpendicular to the radius of the circular motion produces **torque**, given by the Greek letter Tau, τ, lowercase.

Torque can be computed with the equation, $\mathbf{\tau = F \cdot r \cdot \sin(\theta)}$. This is called the cross-product of two vectors, so $\mathbf{\tau = r \times F}$. Torque is a vector.

In the torque equation, r is the perpendicular distance from the line-of-action of the force and the point of rotation. The angle θ is measured between the force and the distance from the axis.

Torque can either start, stop, or change the direction of rotation and is measured in Newton·meters, Nm. This tells us as we study Rotational Dynamics that the cause of rotation is torque not just force.

The **Moment of Inertia** of an object is a measure of its resistance to changes in rotational motion. Just think of the spin of an ice skater, with arms-out, and then arms tucked in.

We use the variable I to indicate this quantity which is called **Rotational Inertia** or even **Moment of Inertia**. In general, this is given by $\mathbf{I = \Sigma mr^2}$.

For example,

- Solid Disk, $I = \frac{1}{2}mr^2$
- Cylindrical Shell, $I = mr^2$
- Solid Sphere, $I = 2/5\ mr^2$

For an object to be in complete equilibrium, it must be in both rotational and translational equilibrium. For **rotational equilibrium**, the **Clockwise Torque** (CWT) must equal the **Counter Clockwise Torque** (CCWT).

Translational equilibrium implies that the sum of the forces in the x direction is zero, and the sum of the forces in the y direction is also zero.

All rotating objects must:

- Obey Newton's second law
- Possess angular momentum which still operates under the conservation law
- Have rotational kinetic energy which can do work
- Obey the conservation law in the absence of any external forces.

Newton's 2nd Law for rotational motion states that the angular acceleration of an object is directly proportional to the applied torque, but varies inversely with the rotational inertia.

The equation is $\boldsymbol{\tau = I \cdot \alpha}$.

To calculate the **work done by a rotating object**, instead of $W = F \cdot s$, we would use $\boldsymbol{W = \tau \cdot \Delta\theta}$.

This also means that to compute **rotary power** we use the equation, $\boldsymbol{P = \tau \cdot \Delta\theta / \Delta t}$, or $\boldsymbol{P = \tau \cdot \omega_{avg}}$.

Rotational Kinetic Energy can be calculated with the equation, $\mathbf{K = \frac{1}{2} I \cdot \omega^2}$.

Lastly, we can compute **angular momentum** with the equation, $\mathbf{L = I \cdot \omega}$.

The applied **angular impulse** would then of course be, $\mathbf{J = \tau \cdot \Delta t}$.

Here is a sample Rotational Mechanics problem:

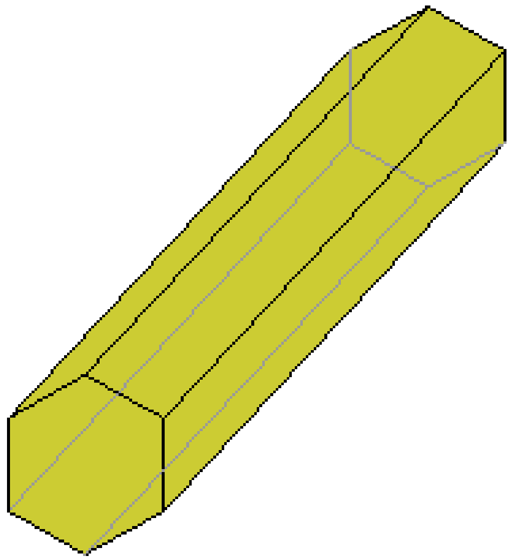
A disk has a rotational inertia of 6.0 kg·m^2 and a constant angular acceleration of 2.0 rad/s^2. If it starts from rest, what is the work done during the first 5.0 s by the net torque acting on it?

Solution: $I = 6.0$ kg·m^2 $\alpha = 2.0$ rad/s^2 $\Delta t = 5.0$ s $\omega_o = 0$ W = _____

$W = \tau \cdot \Delta\theta$ and $\tau = I \cdot \alpha$ To find θ, use the equation, $\Delta\theta = \omega_o \cdot \Delta t + \frac{1}{2}\alpha \cdot (\Delta t)^2$

Therefore, $W = I \cdot \alpha \cdot (\frac{1}{2}\alpha \cdot (\Delta t)^2) = 6 \cdot 2 \cdot (\frac{1}{2} \cdot 2 \cdot (5)^2) =$ **300 J**

Chapter 10 - Equilibrium and Elasticity.



Part of this chapter is concerned with the conditions under which a rigid object is in equilibrium. This term implies either that the object is at rest or that its center of mass moves with constant velocity. We deal here with the former, which are referred to as static equilibrium.

One necessary **condition for equilibrium** is that the net force on an object be zero, $\mathbf{\Sigma F = 0}$. If the object is treated as a particle, this is the only condition that must be satisfied for equilibrium.

It is also good to still know the location of the center of mass, which is the same point as the center of gravity, $\mathbf{x_{cm} = \Sigma(m_i \cdot x_i)/\Sigma m_i}$. (Similarly for y coordinate.)

If the net force on the particle is zero, the particle remains at rest (if originally at rest) or moves with constant velocity (if originally in motion.)

The situation with real (extended) objects is more complex because objects cannot be treated as particles.

In order for an extended object to be in static equilibrium, the net force on it must be zero and it must have no tendency to rotate. This **second condition of equilibrium** requires that the net torque about any origin be zero, $\Sigma\tau = 0$.

In order to establish whether or not an object is in equilibrium, we must know its size and shape, the forces acting on different parts of it, and the points of application of the various forces. Then we can apply the equation, $\tau_{cw} = \tau_{ccw}$.

The last part of this chapter deals with the realistic situation of objects that deform under load conditions. Such deformations are usually elastic innature and do not affect the conditions of equilibrium.

By **elastic** we mean that when the deforming forces are removed, the object returns to its original shape.

Several elastic constants are defined, each corresponding to a different type of deformation.

The **elastic properties of solids** are described in terms of **stress and strain**. Stress is a quantity that is proportional to the force causing a deformation of the object. **Stress = F/A** . (Same as pressure).

Strain is a measure of the degree of the resulting deformation. **Strain =** $\Delta L/L_o$.

The **elastic modulus** of a material is the ratio of stress to strain for that material. There is an elastic modulus for each of the three types of deformation.

Young's modulus which measures resistance to change in length, is given by $Y = (F/A)/(\Delta L/L_o)$.

Shear modulus which measures resistance to relative motion of the planes of a solid, is given by the equation, **$S = (F/A)/(\Delta x/h)$** .

Bulk modulus which measures the resistance to a change in volume, is given by **$B = (\Delta F/A)/(\Delta V/V_o)$** .

Here are some solved problems on static equilibrium and elasticity:

1. Point masses are arranged in the first quadrant of an x-y coordinate system as follows: 6 kg at (.5 m, 2.5 m), 3 kg at (3.5 m, 2.5 m), and 4 kg at (2 m, .5 m). Find the x and y coordinates of the center of mass.

Solution: $m_1 = 6$ kg $x_1 = .5$ m $y_1 = 2.5$ m

$m_2 = 3$ kg $x_2 = 3.5$ m $y_2 = 2.5$ m $m_3 = 4$ kg

$x_3 = 2$ m $y_3 = .5$ m x_{cm} = ______ y_{cm} = ______

$x_{cm} = \Sigma(m_i \cdot x_i)/\Sigma m_i = (m_1 \cdot x_1 + m_2 \cdot x_2 + m_3 \cdot x_3)/(m_1 + m + m_3)$

$x_{cm} = (6kg \cdot .5\ m + 3kg \cdot 3.5\ m + 4\ kg \cdot 2m)/(6kg + 3kg + 4kg)$

$x_{cm} = (3\ m + 10.5\ m + 8\ m)/(13) = 21.5\ m/13 =$ **1.7 m**

$y_{cm} = \Sigma(m_i \cdot y_i)/\Sigma m_i = (m_1 \cdot y_1 + m_2 \cdot y_2 + m_3 \cdot y_3)/(m_1 + m_2 + m_3)$

$y_{cm} = (6kg \cdot 2.5\ m + 3\ kg \cdot 2.5\ m + 4\ kg \cdot .5\ m)/(13\ kg)$

$y_{cm} = (15\ m + 7.5\ m + 2\ m)/(13) = 24.5\ m/13 =$ **1.9 m**

2. A 20 m long steel wire (cross-section 1 cm^2, with a Young's modulus of 2×10^{11} N/m^2), is

subjected to a load of 25,000 N. How much will the wire stretch under the load?

Solution: L_o = 20 m   A = 1 cm^2 = 1 x 10^{-4} m^2

Y = 2 x 10^{11} N/m^2 F = 25,000 N   ΔL = ______

Use the equation, Y = (F/A)/(ΔL/L_o) and solve for ΔL

ΔL = (F·L_o)/(A· Y)

= (25000 N·20 m)/(1 x10^{-4} m^2 · 2 x 10^{11} N/m^2)

ΔL = 2.5 x 10^{-2} m = **2.5 cm**

Chapter 11 - Universal Gravitation.

Much of what we know about universal gravitationis because of the work of the early astronomers and mathematicians. In particular, we must mention the contributions of Copernicus, Brahe, Kepler, Galileo, Newton, and Cavendish.

Nicolaus Copernicus (1473-1543), Poland, suggested that the Earth and all other planets revolve in circular orbits around the Sun, a heliocentric system, not the geocentric model that persisted for 1400 years.

Tycho Brahe (1564-1601), Denmark, charted the positions of the planets and 777 stars for 20 years.

Johannes Kepler (1571-1630), Germany, Brahe's assistant who studied the data from the charts for 16 years and finally formulated 3 laws of planetary motion.

Galileo Galilei (1564-1642), Italy, who perfected the telescope and was later placed under house arrest and forced to recant for supporting the heliocentric theory.

Isaac Newton (1642-1727), England, derived the Law of Universal Gravitation which states that all masses attract each other with a force that varies with the inverse-square of the distance.

Henry Cavendish (1731-1810), England, in 1798 experimentally confirmed the numerical value of the constant in Newton's Inverse-Square Law with a torsion apparatus.

Kepler's 3 Laws of Planetary Motion state the following:

(1) All planets revolve in elliptical, nearly circular, orbits around the Sun.

(2) A straight line from a planet to the sun sweeps out equal areas in equal time intervals.

(3) The cube of the orbital radius of any planet divided by the square of its period is constant. The equation we get is $r^3/T^2 = k$.

From Newton we have the **Law of Universal Gravitation**: "The force of attraction between two bodies is directly proportional to the product of their masses but varies inversely with the square of the distance between them."

The law can be stated as $F_G = Gm_1m_2/r^2$.

The value of the **universal gravitational constant**, **G**, was predicted by Newton to have a value of $6.67x10^{-11}\ Nm^2/kg^2$.

The fact that **gravitational force is centripetal** allows the computation of planetary periods and orbital radii. Recall that $F_C = mv^2/r$ **.**

The mass of the Sun can be found from the period and radius of a planet's orbit. **The Sun's mass is computed to be ≈ 2.0×10^{30} kg**.

The mass of a planet can be found only if it has a satellite orbiting it. For example the **Earth's mass can be calculated to be ≈ 5.98×10^{24} kg**.

A satellite in a circular orbit accelerates centripetally toward Earth at a rate equal to the acceleration of gravity at its orbital radius.

The following **properties of satellite motion** can all be proven:

(1) the velocity is given by the equation $\mathbf{v = 2\pi r/T}$

(2) the acceleration due to gravity at the orbital radius, R, is $\mathbf{g = G \cdot M_E / R^2}$

(2) the minimum or critical velocity for stable orbit is $\mathbf{v = \sqrt{(R \cdot g)}}$.

All bodies have **gravitational fields** around them, which can be represented by a collection of vectors representing the force per unit mass at all locations.

The gravitational field at a point in space is given by the equation: $\mathbf{g = F_G / m}$, measured in N/kg.

Gravitational Potential Energy is given by the equation $\mathbf{U = -Gm_1m_2/r}$.

It is given as negative because the force is attractive, and since U=0 where particle separation is infinite.

Total Energy, E = K + U, can be computed with the equation $\mathbf{E = \frac{1}{2}mv^2 - GMm/r}$.

For elliptical orbits around a massive object this reduces to the form **$E = -GMm/(2a)$** , with **a** equal to the semi-major axis.

To calculate **Escape Velocity**, we have the equation **$v_{esc} = \sqrt{(2GM/R)}$** for a planet of mass, M, and radius, R .

This is derived by using Conservation of Energy, **$\frac{1}{2}mv^2 - GM_Em/R_E = - GM_Em/r_{max}$** by solving for v and letting **$h = r_{max} - R_E$** .

Albert Einstein(1879-1955), proposed that gravity is not a force, but a property of space itself. Mass curves space causing objects to be accelerated toward these massive bodies.

Einstein's theory, called the **General Theory of Relativity**, makes predictions slightly different from Newton's laws, but when tested, gives correct results.

Light has also shown to be deflected by massive celestial objects, and if a mass is large enough, light leaving it will be totally bent back to the object. This predicts that **black holes** in space exist.

Here are some solved sample problems on universal gravitation:

1. A mass of 2.5×10^4 kg is located at a distance of .16 cm from an unknown mass. If the force of attraction between the two masses is measured to be .078 N, find the unknown mass.

Solution: $m_1 = 2.5 \times 10^4$ kg $r = .16$ m $F = .078$ N
$m_2 =$ _____

Using Newton's Gravitation Law, $F_G = G \cdot m_1 m_2/r^2$

$m_2 = (F_G \cdot r^2)/(G \cdot m_1)$

$m_2 = (.078\ N \cdot 1.6 \times 10^{-3}\ m)/(6.67 \times 10^{-11}\ Nm^2/kg^2 \cdot 2.5 \times 10^4\ kg$ = **75 kg**

2. If the mass of the Earth is approximately 5.98 x 10^{24} kg, and the period of the Moon is about 27 days, find the distance from the Earth to the Moon.

Solution: $m_E = 5.98 \times 10^{24}$ kg

$T_M = 27\ da \cdot 24\ hr/da \cdot 3600\ s/hr) = 2.33 \times 10^6\ s$

$r =$ _____

Using Newton's Gravitation Law, $F_G = G \cdot m_{E \cdot} m_M/r^2$ with the fact that gravitational force is centripetal with, $F_G = F_c = m_{M \cdot} v^2/r$, and $v = 2\pi r/T_M$, we get the equation, $G \cdot m_{E \cdot} m_M/r^2 = m_M \cdot v^2/r = m_M \cdot 4\pi^2 r/ T_M{}^2$

$r = (G \cdot m_{E \cdot} T_M{}^2/(4\pi^2))^{(1/3)}$

$r = (6.67 \times 10^{-11}\ Nm^2/kg^2 \cdot 5.98 \times 10^{24}\ kg \cdot (2.33 \times 10^6\ s)^2 /(4\pi^2))^{(1/3)}$ ≈ **383000 km**

Chapter 12 - Oscillatory Motion.

A very special kind of motion occurs when the force on a body is proportional to the displacement of the body from equilibrium.

If this force always acts toward the equilibrium position of the body, there is a repetitive back-and-forth motion about this position.

Such motion is an example of what is called **periodic or oscillatory motion**. You are most likely familiar with several examples of periodic motion, such as the oscillations of a mass on a spring, the motion of a pendulum, and the vibrations of a stringed musical instrument.

Most of the material in this chapter deals with idealized periodic motion, called **Simple Harmonic Motion (SHM)**. In this type of motion, an object oscillates between two spatial positions for an indefinite period of time with no loss in mechanical energy.

In real mechanical systems, retarding (frictional) forces are always present. We call these systems "**Damped Oscillations**." Consequently, in order to maintain their motion they must be "driven", hence the term, "**Forced Oscillations**", later in the chapter.

The most common system which undergoes simple harmonic motion is the **mass-spring system**. The mass is assumed to move on a horizontal, frictionless surface while the spring is fastened to a wall. Or, the spring can be hung from a beam and the mass is attached to the free end of the spring.

The point x = 0 is the **equilibrium position** of the mass; that is, the point where the mass would reside if left undisturbed. In this position, there is no net force on the mass.

When the mass is displaced a distance x from its equilibrium position, the spring produces a linear restoring force given by **Hooke's Law**, **F = -kx**, where k is the force constant of the spring, and has SI units of N/m.

This law is named after **Robert Hooke**, a British scientist and mathematician who lived from 1635 to 1703, and was a contemporary of Isaac Newton.

In simple terms, Hooke's Law states that the force required to stretch a spring is directly proportional to the distance stretched, as long as the elastic limit is not exceeded.

The **minus sign** in **F = -kx** means that F is to the left when the displacement x is positive, whereas F is to the right when x is negative.

In other words, the direction of the force F is always towards the equilibrium position.

If a graph of F versus x is plotted, the slope will be k, the elastic constant. It also should be apparent that the **area under the graph** represents the work done in stretching the spring.

Therefore, **$W = \int F(x)\,dx = \int kx\,dx = ½kx^2$** .

Since energy is the ability to do work, **W = U**, the elastic potential energy of the spring, and **$U = ½kx^2$**.

You should study carefully the comparison between the motion of the mass-spring system and that of the simple pendulum.

A **simple pendulum** consists of a mass m attached to a light string of length L.

In particular, notice that when the displacement is a maximum, the energy of the system is entirely potential energy; whereas, when the displacement is zero, the energy is entirely kinetic energy.

Thisagrees with the fact that **v = 0 when x = A**, the **amplitude**, or maximum displacement from the equilibrium position.

Therefore, it follows that **$v = v_{max}$, when x = 0**.

For an arbitrary value of x, the energy is the sum of K and U.

When the angular displacement is small during the entire motion (**less than about 10 degrees**), the pendulum exhibits SHM. In this case, the resultant force acting on the mass m equals the component of weight tangent to the arc, and has a magnitude **F = mg·sin(θ)**.

Since this force is always directed towards θ = 0, it corresponds to a restoring force. **For small θ, we use the approximation that sin(θ) = θ**.

The period depends only on the length of the pendulum and the acceleration of gravity. The period does not depend on mass, so we conclude that all simple pendula of equal length oscillate with the same frequency, f, and period, T.

For the simple pendulum $\mathbf{T = 2\pi\sqrt{(L/g)}}$, while other pendula have equation $\mathbf{T = 2\pi\sqrt{(I/mgL)}}$.

Period and frequency are reciprocal, $\mathbf{T = 1/f}$. And using the equations $\mathbf{v = 2\pi r/T}$ and $\mathbf{v = \omega r}$, we can show that $\mathbf{\omega= 2\pi f}$. We call $\boldsymbol{\omega}$ the **angular frequency**.

Similarly, it can be shown that the maximum speed of an object in SHM is given by $\mathbf{v_{max} = A\omega}$. Also the maximum acceleration is, $\mathbf{a_{max} = A\omega^2}$.

Another way to express angular frequency is with $\mathbf{\omega = \sqrt{(k/m)}}$.

The position x of a simple harmonic oscillator varies periodically with time according to the expression $\mathbf{x(t) = A\cos(\omega t+\varphi)}$.

In the previous equation, **A** is the **amplitude** of the motion, $\boldsymbol{\omega}$ is the **angular frequency**, and $\boldsymbol{\varphi}$ is the **phase constant**.

The velocity of a simple harmonic oscillator is given by $\mathbf{v(t) = -\omega A\sin(\omega t+\varphi)}$.

The acceleration is $\mathbf{a(t) = -\omega^2 A\cos(\omega t+\varphi)}$. It also can be shown that $\mathbf{v = \pm\omega\sqrt{(A^2-x^2)}}$.

The Kinetic and Potential Energy of a simple harmonic oscillator vary with time and are given by $\mathbf{K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2A^2\sin^2(\omega t+\varphi)}$ and the equation $\mathbf{U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t+\varphi)}$. The Total Energy is $\mathbf{E = \frac{1}{2}kA^2}$.

If an oscillator experiences damping, then **R** = -b**v**.

Its position for small damping is given by the equation $\mathbf{x = Ae^{-(b/2m)t}\cos(\omega t+\varphi)}$, where we now have $\boldsymbol{\omega = \sqrt{(k/m - (b/2m)^2)}}$.

If the oscillator is subject to a sinusoidal driving force $\mathbf{F(t) = F_o \sin(\omega t)}$, it exhibits resonance, in which the amplitude is largest when the driving frequency matches the natural frequency of the oscillator.

Here are some solved sample problems in oscillatory motion:

1. A body oscillates with simple harmonic motion along the *x*-axis. Its displacement varies with time according to $x = 5.0 \sin(\pi t + \pi/3)$. What is the acceleration (in m/s^2) of the body at t = 1.0 s ?

Solution: $x = 5.0 \sin(\pi t + \pi/3)$

$dx/dt = v = 5.0\pi \cos(\pi t + \pi/3)$

$dv/dt = a = -5.0\pi^2(\sin(\pi t + \pi/3)$

at t = 1.0 s , $a = -5.0\pi^2(\sin(\pi t + \pi/3)$ = $\mathbf{-43\ m/s^2}$

2. A block of mass 4 kg is attached to a spring, and undergoes simple harmonic motion with a period of T = .35 s. The total energy of the system is E = 2.5 J. What is the force constant of the spring? What is the amplitude of the motion?

Solution: First, find the angular frequency of the motion given by $\omega = 2\pi/T = 2\pi/.35\text{ s} = 18\text{ rad/s}$

Now $\omega = \sqrt{(k/m)}$

Therefore, $k = m\cdot\omega^2 = 4\text{ kg}\cdot(18\text{ rad/s})^2 =$ **1300 N/m**

Total energy is $E = ½kA^2$

$A = \sqrt{(2E/k)} = \sqrt{(2\cdot 2.5\text{ J}/1300\text{ N/m})} =$ **.062 m**

Appendix

List of Physical Constants

Universal gravitational constant , G = 6.67×10^{-11} Nm^2/kg^2
Acceleration due to gravity, g= 9.81 m/s^2
Speed of light in a vacuum, c = 3.00×10^8 m/s
Speed of sound in air at STP = 331.5 m/s
Mass of Earth = 5.98×10^{24} kg
Mass of the Moon = 7.35×10^{22} kg
Mean radius of Earth = 6.37×10^6 m
Mean radius of the Moon = 1.74×10^6 m
Mean distance, Earth to the Moon = 3.84×10^8 m
Mean distance, Earth to the Sun = 1.50×10^{11} m
Rest mass of the electron, $m_e = 9.11 \times 10^{-31}$ kg
Rest mass of the proton, $m_p = 1.67 \times 10^{-27}$ kg
Rest mass of the neutron, $m_n = 1.67 \times 10^{-27}$ kg

Approximate Coefficients of Friction

	Kinetic	**Static**
Rubber on concrete (dry)	0.68	0.90
Rubber on concrete (wet)	0.58	
Rubber on ice	0.15	
Waxed ski on snow	0.05	0.14
Wood on wood	0.30	0.42
Steel on steel	0.57	0.74
Copper on steel	0.36	0.53
Teflon on Teflon	0.04	

Prefixes for Powers of 10

Prefix	**Symbol**	**Notation**
tera	T	10^{12}
giga	G	10^9
mega	M	10^6
kilo	k	10^3
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	µ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}